# Ranking teams in partially-disjoint tournaments

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## 1 Introduction

Throughout sports, whether it is professional or collegiate sports, teams are ranked. In professional sports, most teams play each other, so it is easier to rank based on winning percentage the teams. However, in collegiate sports, there are more issues because most teams do not play each other, so teams cannot be ranked solely based on winning percentage. Two rankings in collegiate sports that are not based solely on winning percentage are RPI and Colley rankings. The problem with NCAA sports is that teams play a lot of games within their conferences, but fewer games outside their conferences. When teams do not play each other it is hard to say with certainty that one team is better than the other. It is postulated by some people that certain teams benefit from playing a lot of in conference games and few out of conference games. One of these people that proposed this is Keegan Cook who was a Saint Mary's College assistant volleyball coach who now coaches for University of Washington. Keegan proposed that teams at the beginning of the season are ranked higher because of their stronger conference, and as the season went along that it was harder for a lower ranked team to rise up the rankings because the higher ranked team and lower ranked team did not play each other. Also since teams do not play very many out of conference games, it is hard to compare teams that basically have no opponents in common. Because of this proposal, we wished to look at this problem and determine if two of the current ranking systems that are used in NCAA sports do well when teams rarely play other teams in other conferences.

## 2 Rankings

The two rankings that we looked at were RPI, which stands for Rating Percentage Index, and the Colley Matrix. RPI is the ranking that is used for most sports in college. The Colley Matrix is one of six computer rankings that the BCS uses to rank teams for college football.

### 2.1 RPI Rankings

The formula for RPI is: $0.25wp + 0.5owp + 0.25oowp$, where $wp$ is the winning percentage of the team, $owp$ is their opponent's winning percentage, and $oowp$ is their opponent's opponent's winning percentage. In order to calculate RPI for our different scenarios, we created a computer program in order to calculate RPI rankings quickly. In order to calculate the winning percentage of each team, we put the wins of each team into a matrix and then multiplied this "win matrix" by the "reciprocal matrix", which is a matrix composed of the reciprocal of the number of games each team played. In order to calculate a team's opponent's winning percentage, we multiplied the "games matrix", which is a matrix composed of which teams played each other, by the winning percentage matrix, and then the "reciprocal matrix" In order to calculate a team's opponent's opponent's winning percentage, we had the team's opponent's winning percentage multiplied by the "games matrix" and then multiplied by the "reciprocal matrix." Then in order to calculate RPI, we used these matrices to put into the equation $0.25wp + 0.5owp + 0.25oowp$.

Here is a small example of how the RPI rankings were calculated in the program:
Let's say that Team A plays 3 total games and wins 3 games, Team B plays 3 total games and wins 2 games, Team C plays 3 total games and wins 1 game, and Team D plays 3 total games and wins 0 games.

The "win matrix" will look like this: $$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

The total number of a wins for a team will be the sum of the row of that team. So in our example Team A has a total number of 3 wins. The 1 in an entry means that a team beats another team, so in our example Team A beats Teams B, C, and D.

The "reciprocal matrix" will look like this: $$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} \end{array}$$

The diagonal of this matrix is the reciprocal of the total number of games each team plays.

The "games matrix" will look like this: $$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

The 1 in an entry means that a team plays another team once. For example, Team A plays Teams B, C, and D.

The win percentage matrix is "win matrix" times "reciprocal matrix".
To calculate opponent's winning percentage matrix: "games matrix" times win percentage matrix times "reciprocal matrix".
To calculate opponent's opponent's winning percentage : opponent's winning percentage matrix times "games matrix" times "reciprocal matrix".

## 2.2 Colley Matrix Rankings

The formula for the Colley Matrix rankings is: $C\vec{r} = \vec{b}$, where $C$ is the Colley Matrix, $\vec{r}$ is a column-vector of the rankings, and $\vec{b}$ is a column vector shown below. The entries for the Colley Matrix, $C$, are $c_{ii} = 2 + n_{tot,i}$ and $c_{ij} = -n_{j,i}$, where $n_{tot,i}$ is the total number of games team $i$ plays and $n_{j,i}$ is the number of times team $i$ plays team $j$. The entries in column vector $b$, $\vec{b}$, are $b_i = 1 + (n_{w,i} - n_{l,i})/2$, where $n_{w,i}$ is the number of wins team $i$ has and $n_{l,i}$ is the number of losses team $i$ has. In order to solve for the rankings for the Colley Matrix, we used a function that uses row reduction, or known as Gaussian elimination, within the programming language to solve for $\vec{r}$, the rankings.

# 3 Complete Dominance

In order to understand how each of the rankings worked, we decided to look at the number of games played between teams in the two different leagues, which we call links. We looked at how many links it would take for one league to be completely ranked higher than the other league, so for one league to be completely dominant over the other. We looked at a total of eight teams, four teams in each league; ten teams, five teams in each league; twelve teams, six in each league; fourteen teams, seven in each league; and sixteen teams, eight in each league.

## 3.1 Eight teams

For a total of eight teams, we had Teams A, B, C, and D in one league and Teams I, J, K, L in the other league.

### 3.1.1 4 Links

For our 4 links, we had all the possible combinations of Teams A-D beating Teams I-L, but each row and column must each sum to 1.

An example of a total of 4 links:
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} I & J & K & L \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

In this example, Team A beats Team J, Team B beats Team K, Team C beats Team L, and Team D beats Team I. Here, the columns and rows each sum to 1.

For one league to be dominant over the other for RPI, Team A cannot beat Team I, so entry 1, 1 in our matrix must be 0. Also Team D cannot beat Team L, so entry 4, 4 in our matrix must be 0. For Colley rankings, 4 links does not make one league completely dominant over the other.

### 3.1.2 8 Links

For a total of 8 links, we had all the possible combinations of Teams A-D beating Teams I-L twice, but each row and column must each sum to 2. An example of a total of 8 links:
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} I & J & K & L \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Here, the columns and rows each sum to 2.

For the Colley rankings to show one league's dominance, Team D had to beat Teams I and J, but there was one exception in which Team D beat Teams I and K. Also Team A cannot beat Team I and Team D has to beat Team I. Either Team B beats Team I or Team C beats Team I. Either Team D beats Team J or Team D beats Team K.

## 3.2 Ten teams

For a total of ten teams, we had Teams A, B, C, D, and E in one league and Teams I, J, K, L, and M in the other league.

### 3.2.1 5 Links

For a total of 5 links, we had all the possible combinations of Teams A-E beating Teams I-M, but each row and column must each sum to 1.

An example of a total of 5 links: $\begin{array}{c c} & \begin{array}{ccccc} I & J & K & L & M \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$

Here, the columns and rows each sum to 1.

For RPI, Team A cannot beat Team I and Team E cannot beat Team L and M. For Colley, one league does not dominate the other.

### 3.2.2 10 Links

For a total of 10 links, we had all the possible combinations of Teams A-E beating Teams I-M twice, but each row and column must each sum to 2.

An example of a total of 10 links: $\begin{array}{c c} & \begin{array}{ccccc} I & J & K & L & M \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$

Here, the columns and rows each sum to 2.

For Colley, one league does not dominate the other.

## 3.3 Twelve teams

For a total of twelve teams, we had Teams A, B, C, D, E, and F in one league and Teams I, J, K, L, M, and N in the other league.

#### 3.3.1 6 Links

For a total of 6 links, we had all the possible combinations of Teams A-F beating Teams I-N, but each row and column must each sum to 1. For RPI, Team F has to beat Team I. Team A cannot beat Team I and Team J. Only Team H can beat Team I, so no other team beats Team I. For Colley, one league does not dominate the other.

#### 3.3.2 12 Links

For a total of 12 links, we had all the possible combinations of Teams A-F beating Teams I-N, but each row and column must each sum to 2. For Colley, one league does not dominate the other.

### 3.4 Fourteen teams

For a total of fourteen teams, we had Teams A, B, C, D, E, F, and G in one league and Teams I, J, K, L, M, N, and O in the other league.

For a total of 7 links, we had all the possible combinations of Teams A-G beating Teams I-O, but each row and column must each sum to 1. For both RPI and Colley, one league does not dominate the other.

### 3.5 Sixteen teams

For a total of sixteen teams, we had Teams A, B, C, D, E, F, G, and H in one league and Teams I, J, K, L, M, N, O, and P in the other league.

For a total of 8 links, we had all the possible combinations of Teams A-H beating Teams I-P, but each row and column must each sum to 1. For both RPI and Colley, one league does not dominate the other.

## 4 Sixteen Team Tournament Simulation

### 4.1 Tournament Set Up

In order to see how the RPI and Colley rankings did under different scenarios, we created a sixteen team tournament simulation. For our sixteen team tournament simulation we split the sixteen teams in two leagues, East and West, where Teams A-H make up the East and Teams I-P make up the West. The tournament consisted of a series of games. These games consisted of every team in the East playing each other once and every team in the West playing each other once. The games in the tournament also included teams from the East playing selected teams from the West, which we called links. These links represent the few games that teams play between different conferences in the NCAA on a smaller scale. We also had the teams have different scores, which denoted their abilities. By giving the teams different scores, we could create different scenarios of leagues. We also weighted the games between the two leagues differently. The weights of these games represented the importance of the game between the two leagues.

### 4.1.1 Links

We created a total 12 different combination of links, which we labeled: 2A, 2B, 2C, 2D, 2E, 2F, 2G, 3A, 3B, 3C, 4A, and 4B Links. The way that we set up the links was using a matrix to show which team from the East played the selected team from the West. The way that we showed a team played another team in the matrix is if there is a 1 in the matrix.

The matrix for 2A Links:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{matrix} I & J & K & L & M & N & O & P \end{matrix} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

In this example, Team A plays Teams I and J and so on.

The matrix for 2B Links is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{matrix} I & J & K & L & M & N & O & P \end{matrix} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The matrix for 2C Links is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{matrix} I & J & K & L & M & N & O & P \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The matrix for 2D Links is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccccccc} I & J & K & L & M & N & O & P \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The matrix for 2E Links is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccccccc} I & J & K & L & M & N & O & P \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

The matrix for 2F Links is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccccccc} I & J & K & L & M & N & O & P \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

The matrix for 2G Links is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccccccc} I & J & K & L & M & N & O & P \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

The matrix for 3A Links is:

$$\begin{array}{c|cccccccc} & I & J & K & L & M & N & O & P \\ \hline A & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ B & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ E & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ F & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ G & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ H & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

The matrix for 3B Links is:

$$\begin{array}{c|cccccccc} & I & J & K & L & M & N & O & P \\ \hline A & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ B & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ C & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ D & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ E & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ F & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ G & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ H & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$

The matrix for 3C Links is:

$$\begin{array}{c|cccccccc} & I & J & K & L & M & N & O & P \\ \hline A & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ B & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ C & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ D & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ E & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ F & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ G & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ H & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{array}$$

The matrix for 4A Links is:

$$\begin{array}{c|cccccccc} & I & J & K & L & M & N & O & P \\ \hline A & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ B & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ C & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ D & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ E & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ F & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ G & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ H & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

The matrix for 4B Links is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array} \begin{array}{c} \begin{array}{cccccccc} I & J & K & L & M & N & O & P \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

### 4.1.2 Leagues

We came up with different scenarios for teams scores in order to simulate different leagues that may occur in college sports. The different leagues that we created were Dominant Team, Dominant League, Top Heavy, Split League 2, Weak Team, Interweave, and Tied Team. For Dominant Team, we had one team that had a score much greater than all the teams. We picked this league because the Dominant Team represents a team in a conference that is much better than all the other teams in a conference. For Dominant League, we had one all the teams in one league with higher scores than all the teams in other league. This league represents how one conference can be better than another conference in college sports. For Top Heavy, one league is sandwiched between the first and last four teams in the other league. This represents how one conference can have really good teams but also have really bad teams. For Split League 2, the leagues are split in half and alternately ranked. This represents how conferences are split in half and have good teams and bad teams. For Weak Team, one team has a much lower score than the rest of the teams in both leagues. This represents how one conference can have a really bad team that wins no games. For Interweave, the teams' scores alternate. This represents how two different conferences can be similarly matched, but the teams in one conference are still better than the corresponding teams in the other conference. For Tied Team, the teams' scores alternate, but two teams in the middle of the different leagues have a tied score. This represents how teams in different conferences can have a similar ability.

| | Dominant Team | Dominant League | Top Heavy | Split League 2 | Weak Team | Interweave | Tied Team |
|---|---|---|---|---|---|---|---|
| Team A | 100 | 32 | 32 | 32 | 92 | 32 | 32 |
| Team B | 28 | 30 | 30 | 30 | 88 | 28 | 28 |
| Team C | 24 | 28 | 28 | 28 | 84 | 24 | 24 |
| Team D | 20 | 26 | 26 | 26 | 80 | 20 | 19 |
| Team E | 16 | 24 | 8 | 16 | 76 | 16 | 16 |
| Team F | 12 | 22 | 6 | 14 | 72 | 12 | 12 |
| Team G | 8 | 20 | 4 | 12 | 68 | 8 | 8 |
| Team H | 4 | 18 | 2 | 10 | 64 | 4 | 4 |
| Team I | 30 | 16 | 24 | 24 | 90 | 30 | 30 |
| Team J | 26 | 14 | 22 | 22 | 86 | 26 | 26 |
| Team K | 22 | 12 | 20 | 20 | 82 | 22 | 22 |
| Team L | 18 | 10 | 18 | 18 | 78 | 18 | 19 |
| Team M | 14 | 8 | 16 | 8 | 74 | 14 | 14 |
| Team N | 10 | 6 | 14 | 6 | 70 | 10 | 10 |
| Team O | 6 | 4 | 12 | 4 | 66 | 6 | 6 |
| Team P | 2 | 2 | 10 | 2 | 2 | 2 | 2 |

### 4.1.3 Weighted Links

When we inputted the games into our program, they were given a regular value of 1. As well as having each game given a value of 1, we decided to weight the links between the leagues. These weights represented the importance of the games between East and West. The link weights that we tested were $0.5, 0.8, 1.2, 1.5$. The link weights of 0.5 and 0.8 represent that the games between the leagues are not as important as the games within the leagues. The link weights of 1.2 and 1.5 represent that the games between the leagues are more important than the games within the leagues. We tested different weights to see if there was an effect on the rankings and see if the weighted links made the rankings better.

## 4.2 Program Overview

The program runs a tournament with each scenario 10,000 times. Each time that tournament is simulated, the rankings of each team are then recorded for RPI and Colley rankings. Once the program has run all the simulations, the program prints out the average ranking of each team for RPI and Colley.

## 4.3 Game Simulation

The way that a game is simulated in the program is by using the teams' scores. From these scores, we can calculate the probabilities of each team winning a game against another team. The way we calculate a team's probability of winning is to take a team's score and divide it by the sum of that same team's score and another team's score. Once we have this probability, a team wins a game if the random number between 0 and 1 that Python generates is less than their probability of winning, and if the random number is greater than their probability of winning, then that teams loses the game.

An example of the game simulation is if Team A has a score of 30 and Team H has a score of 10, then the probability of Team A winning the game against Team H is: $\frac{30}{(10+30)} = 0.75$. Team A wins the game if the random number generated by Python is less than 0.75. If the random number generated is greater than 0.75, then Team A loses and Team H wins.

## 4.4 Best Fit Rankings

We used the Chi Squared Test to see how the observed rankings, the average of the RPI and Colley rankings from the program, compared with the expected rankings, the known rankings based on the scores we gave the teams. We used the Chi Squared Test because is the goodness of fit test and we wanted to see how well the observed rankings did compared to the expected rankings. The formula for the Chi Squared Test is: $\chi^2 = \sum \frac{(o-e)^2}{e}$, where $o$ is the observed rankings and $e$ is the expected rankings. The critical value for the Chi Squared Test for our data is 2.038. The critical value is based on 15 degrees of freedom and a 99% confidence interval. If the calculated chi squared value is than 2.038, then with 99% confidence we can say that the observed ranking is a good fit and a good ranking of the teams. If the calculated chi square number is greater than 2.038, then with 99% confidence we can say that the observed ranking is not a good fit and is not a good ranking of the teams.

## 4.5 Results

After running the different scenarios in the program, we inputted the observed rankings in an Excel spreadsheet in order to calculate the chi squared values.

### 4.5.1 Link Weight of 1

Here is a table for RPI Good Fit Rankings, Chi squared values less than 2.038:

| | Dominant Team | Dominant League | Top Heavy | Split League 2 | Weak Team | Interweave | Tied Team |
|---|---|---|---|---|---|---|---|
| 2A Links | | X | | X | | | |
| 2B Links | | X | | X | | | |
| 2C Links | | X | | X | | | |
| 2D Links | X | X | | X | | X | X |
| 2E Links | X | | | | | | X |
| 2F Links | | | | | | | |
| 2G Links | X | | | | | | |
| 3A Links | | X | | X | | | |
| 3B Links | | X | | X | | X | X |
| 3C Links | | X | | | | | X |
| 4A Links | | | | X | | | |
| 4B Links | X | X | X | X | | X | X |

An "X" in the table means that the observed rankings were good fits for the expected rankings for the different links and leagues. If there is a blank space in the table means the observed rankings were not good fits for the expected rankings for the different links and leagues.

For Weak Team, the RPI rankings were not good fits for all the different links. The RPI rankings were also not good fits for Top Heavy for all of the different links except one of the links. We found that the Split League 2 RPI rankings were good fits for 67% of the different links. We found that the Interweave RPI rankings were good fits for 25% of the different links. The Dominant Team RPI rankings were good fits for 33% of the different links. The Dominant League RPI rankings were good fits for 66% of the different links. The Tied Team RPI rankings were good fits for about 50% of the different links.

For 2F Links, all the RPI rankings for the different leagues were not good fits. For RPI rankings, four of the different leagues, Weak Team, Top Heavy, Interweave, and Dominant Team, were not good fits for the majority of the different links. There were only two leagues, Split League 2 and Dominant League, to have a good RPI ranking fit for the majority of the different links. The Tied Team had about half of the links that were good and not good fits for the RPI rankings. Overall, the RPI rankings for our different leagues and links were not good fits for their expected rankings.

For the Colley rankings, all the leagues, Weak Team, Top Heavy, Split League 2, Interweave, Dominant Team, Dominant League, and Tied Team, and links were good fits for the expected rankings. For the Tied Team league, both RPI and Colley had either Team D or Team L ranked higher than the other instead of tied in rankings.

### 4.5.2 Other Link Weights

For 2B Links and link weights of 1.2, the observed rankings for all the leagues could not be calculated because when solving for the Colley rankings the output was a singular matrix, which means that the matrix cannot be solved. We found that when we weighted the links there were certain links where all of the leagues had lower chi squared values than chi squared values of the link weight

1 or higher chi squared values than chi squared values of the link weight 1. When all the leagues for certain links had lower chi squared values, the link weights were either 0.5 or 0.8. When all the leagues for certain links had higher chi squared values, the link weights were either 1.2 or 1.5. This is interesting because most college sports fans and college coaches think that out of conference or league games are at times more important and will help their rankings.

# 5 Concluding Thoughts

Overall, we found that the Colley rankings did better in ranking teams than the RPI rankings. The Colley rankings did better because the observed rankings were good fits for all the different leagues and links, while the RPI rankings were good fits for about 34.5% of all the different leagues and links.

The downfall of the RPI rankings is that it does not rank teams well when good teams play bad teams. The RPI rankings actually hurt a good team if they played a bad team. If a good team played a bad team, the good team would be ranked lower than expected.

For Tied Team, the results were inconclusive because the teams that were tied did not have tied rankings. Instead, one team was ranked higher than the other, and this was dependent on which links were tested to determine which of these teams were ranked higher. We would have to further examine what makes one tied team in one league ranked higher than another tied team in another league, whether that be what league they are in, a strong or weak league, or who each of the teams play. This scenario of the Tied Team is interesting because throughout collegiate sports there are a lot of teams in the sports that have the same capability and are essentially evenly matched. It would be interesting to see if by playing in a strong conference and playing stronger teams would help a tied team be ranked higher.

# 6 References

Colley, W. N. (2002) Colleys bias free college football ranking method: the Colley matrix explained.

# 7 Acknowledgments

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