

Photon Statistics

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The purpose of this experiment is to determine the probability distribution of photons upon arrival from a monochromatic laser light source and a thermal light source. A green diode laser was used as both sources, fired directly to act as the monochromatic source and fired through spinning glass to mimic the effects of a thermal source. This pseudo-thermal source had to be used instead of an actual thermal source due to the small coherence time of an actual thermal source. It was concluded that the monochromatic laser source follows a Poisson distribution while a pseudo-thermal source follows a Bose-Einstein distribution. In addition, it appears that Poisson is the default distribution for light sources while Bose-Einstein is the singular case.

I. Introduction

Classically, one views light as a continuous beam where energy is extractable in any amount. However, upon the introduction of the quantum theory, we know that light consists of small, quantized particles called photons and when summed together make up the light beam. These particles are not of any energy but consist of specific energies and only integer steps of these energies can be extracted. For example: a monochromatic light consisting of only the wavelength λ has $E = nh\nu$ as the amount of extractable energy, where n is the number of photons, h is Planck's constant and ν is the wave frequency. The average intensity of a beam of light, either from a laser, a flame, a light bulb, etc, is proportional to the photon flux, therefore the more intense the beam the more photons n that can be extracted. My experiment is to measure two such sources: a laser source and a thermal source, and find a discernable pattern of how the photons are emitted and see if it matches a hypothesized pattern.

It is hypothesized that light arrives at its destination about an average due to its probabilistic properties. These properties involve the mysterious quantum particle-wave duality where quantized particles, such as photons, act as waves with a probability of being at a certain location at a specific time: but once measured act as particles. For a beam of constant intensity the probability distribution of photons follows a Poisson distribution. For light of variable intensity, such as a thermal source, the photon distribution follows the Bose-Einstein distribution.

To conduct the experiment I used a monochromatic laser to act as both sources of light. By directing the laser straight into a photomultiplier tube a Poisson distribution was achieved. For the thermal source we set up the same experiment but directed the laser through a spinning piece of ground glass. The reason an ideal thermal source could not be used is because of the small coherence time property that is exhibited by the photons. The pseudo-thermal light mimics the variable effects of a thermal source via the constructive and destructive interference caused by the speckle of the ground glass, thus leading to a Bose-Einstein distribution.

II. Background

Since the sixties scientists have been counting photons using similar techniques to those explained below. It is via these experiments and a little calculus that one can hypothesize both distributions found in monochromatic laser and thermal light sources. Before breaking into the two specific distributions well look at what holds true for light in general. To begin we will first derive the intensity (W) of integrated light¹.

¹P. Koczyk, American Association of Physics Teachers, 1996

$$W = \int \int_A \int_t^{t+T} I(x, y; \xi) d\xi dx dy \quad (1)$$

$I(x, y, \xi)$ is the intensity of the wave at point (x, y) and time ξ . The limits of integration are time t to $t+T$ where T is the time step and the area A for the illuminated area of the photomultiplier tube. This intensity W is a stochastic variable, it randomly fluctuates, meaning W resembles a probability density function $P(W)$. In this case $P(W) = \delta(W - \bar{W})$ where \bar{W} is the mean intensity (known because it is then intensity the laser is rated for.) With this we can solve for the probability of observing n counts using Mandels formula²,

$$P(n) = \int_0^\infty \frac{(\alpha W)^n}{n!} e^{-\alpha W} P(W) dW \quad (2)$$

where the limits of integration are 0 to ∞ and $\alpha = \frac{\eta}{h\nu}$ where η is the photomultiplier tubes quantum efficiency, h is Plancks constant, and ν is the light wave frequency. From here we solve the delta function $P(W) = \delta(W - \bar{W})$ in equation (2). After solving the delta function we see αW becomes \bar{n} , the average count, and the equation becomes the Poisson equation

$$P(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \quad (3)$$

This distribution is for a light source where individual photons are independent of one another and the source is rated at an average intensity. In our second case light isnt fired from a source at an average rate, \bar{W} . The thermal light field is an example where light fired is a superposition of many wave states with random amplitudes and phases, caused by the exciting of individual atoms randomly and their following discharge. This is where the coherence time property comes into play.

For a thermal light source the light field changes over time, due to the change in phases and amplitude of the wave states (this is what the spinning ground glass mimics). This makes predicting the count over a specific time somewhat impossible, each photon becomes essentially independent and the distribution slips into the default Poisson distribution. However, for a small amount of time the field is correlated and each photon is somewhat connected. This small amount of time is the coherence time, and when data is taken in a time step sufficiently smaller than this time the distribution will follow the Bose-Einstein distribution.

This coherence time for a true thermal source is very short, typically less than $1ps^3$, making it extremely difficult to measure with counting techniques. The laser coherence time is much larger, thus combining this property with the random surface imperfections (speckle) of ground glass culminates to the emitted pattern of the pseudo-thermal light source being very similar to an actual thermal source whilst still allowing for a larger time step to count with.

For the thermal source we use Mendels formula once more, but instead of the independent photon intensity model $P(W)$ in equation (1) we use⁴

$$P(I) = \frac{1}{I} e^{-\frac{I}{T}} \quad (4)$$

²P. Koczyk, American Association of Physics Teachers, 1996

³Loudon, The Quantum Theory of Light, 1983

⁴P. Koczyk, American Association of Physics Teachers, 1996

where \bar{I} is the average intensity. Plugging this in for $P(W)$ in equation (2) and simplifying we get

$$P(n) = \frac{\alpha^n}{n!\bar{I}} \int_0^\infty I^n e^{-I\alpha + \frac{-I}{\bar{I}}} dI \quad (5)$$

where the limits of integration are 0 to ∞ . To solve for $P(n)$ we use an integration table with the property:

$$\int x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \quad (6)$$

With this property we can solve for $P(n)$, remember $\alpha\bar{I} = \bar{n}$ (the average number of counts). The final answer comes to the Bose- Einstein distribution

$$P(n) = \frac{1}{1 + \bar{n}} \frac{\bar{n}^n}{(1 + \bar{n})^n} \quad (7)$$

The final statistical data that is needed for this experiment is the variations for each distribution. Using the Poisson distribution data and the definition of variance we know that the variance of a Poisson fit should be equal to the mean: $\sigma^2 = \bar{n}$. Using the Bose-Einstein distribution and the definition of variance it is determined that the variance for a Bose-Einstein fit should be equal to the mean squared plus the mean; $\sigma^2 = \bar{n}^2 + \bar{n}$. Comparing these two variances we see that a Bose-Einstein fit has more fluctuation in the photon count than the Poisson fit.

With the variance known we can also calculate the signal to noise ratio (SNR). This is important because it gives us the ratio of a wanted number of photons, the signal, to the unwanted number of photons, the noise. This is important in a case where the signal is carrying information, the more signal there is the more successful the transfer of information will be. To calculate the signal to noise ratio one simply divides the mean squared by the variance; $\frac{(\bar{n})^2}{\sigma^2}$. The SNR for Poisson is the mean; \bar{n} , while the SNR for Bose-Einstein is $\frac{(\bar{n})}{(\bar{n}+1)}$. This shows that light following the Poisson distribution is more suitable for transferring data than light following the Bose-Einstein distribution.

III. Experiment

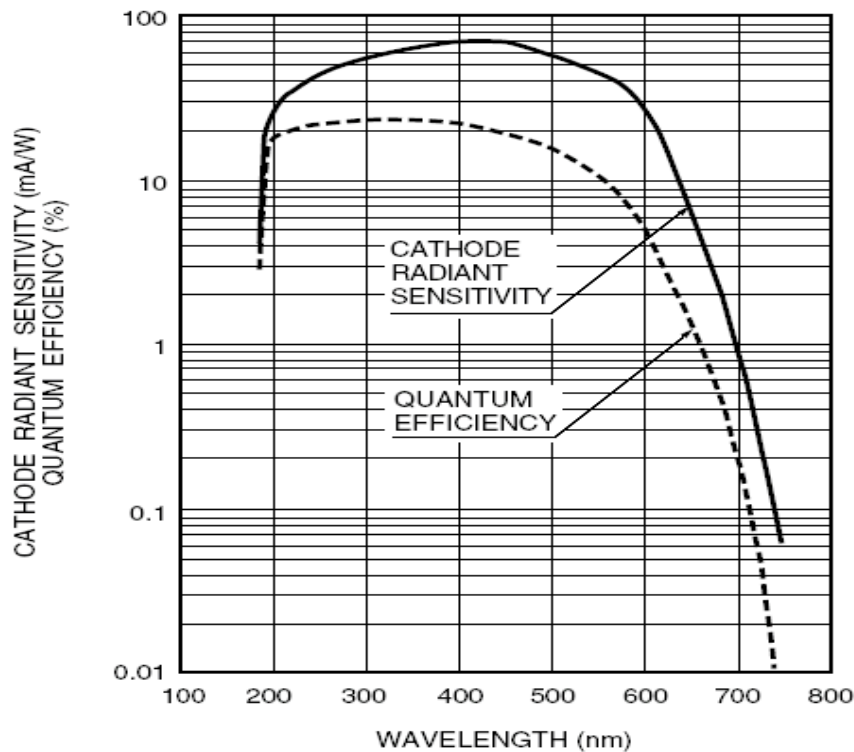
Fig. 1 Experiment Setup

Laser → Ground Glass → PMT → Amplifier → Discriminator → Counter → Computer → Labview → Excel

To begin our experiment we had to choose a suitable light source, one that can act as both a monochromatic and a thermal source. We choose to use a simple Pasco OS-8458 Diode Laser with a wavelength of 532nm because of its proximity to the maximum quantum efficiency of the photomultiplier tube.

For the monochromatic data we directed the laser at the photomultiplier tube (PMT). To bring the intensity down and get an accurate reading within the range of the PMT we typically used two polarizers, set at a specific angle between them. For the pseudo-thermal source we fired the same laser through a spinning piece of 2-inch diameter ground glass attached to a DC motor. Again, after the light exited the glass, we typically used two polarizers.

⁵Feynman, The Feynman Lectures on Physics, 1965



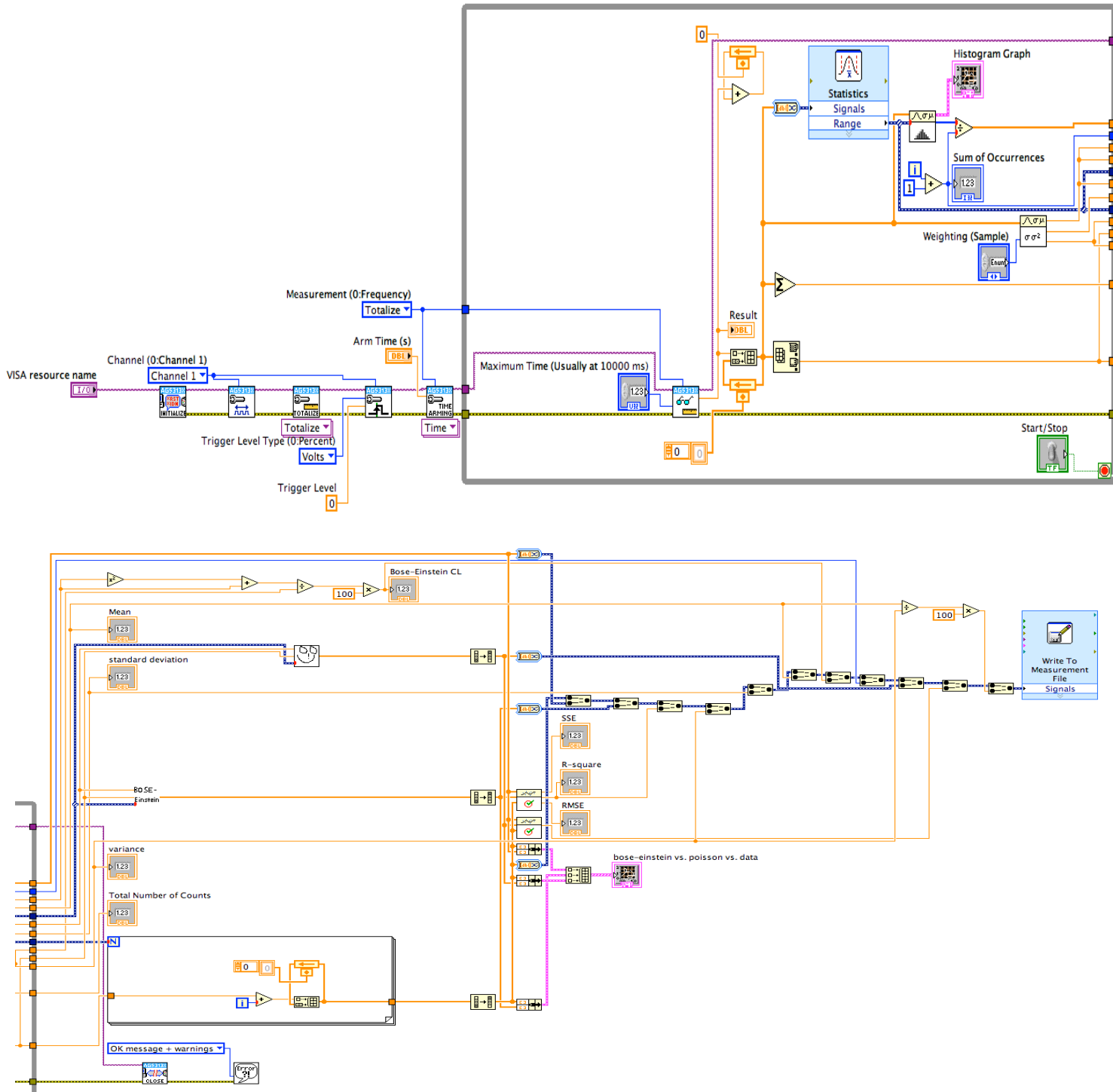
The current from the PMT was then sent into a Lecroy 612A amplifier via BNC cable, multiplying the entrance current by ten. This was important because it allowed for the discriminator to easily distinguish dark current from regular current. The exiting current was then sent into a Lecroy 821 Quad Discriminator. With the discriminator we could adjust the pulse width of the current as well as the incident threshold. The threshold was set to a voltage that allowed a full charge to be sent into the counter but halted those below the full charge, the dark current. This was around 1 volt, meaning any pulse entering the amplifier that was less than 100mV was determined to be dark current. Whenever a charge above the threshold was read by the discriminator a red LED light would light up showing a count. The discriminator changed the signal of the pulse from various sized spikes due to where in the incident photon struck the PMT to a uniform square current with a certain amplitude and pulse width. This width was adjusted to 250ns, the smallest allowed width keeping the square pulse intact and maximizing the number of counts that could be taken in a given time; the amplitude was set to 8 volts.

From the discriminator the current went into a HP 53131A Universal Counter controlled by the program Labview via a computer. The counters job was to count and record each blip from the current. The counter is designed to count square waves, thus if the discriminator sent out a pulse that was not square the counter could send out more than one count at a time. One could set the arm time on the counter, meaning it counted for a specific set time, recorded it, then went on to the next time step. The counter continued this until told to stop. Graphically analyzing these sets of data led to our finalized data. The counters arm time was generally set to the smallest possible time, 1ms.

The end of this information line is found at the computer screen via Labview. With this program we read the data from the counter, graphed it, and compared it to the hypothesis. Programming the Labview program used was a project in itself and wont be explained in detail. Simply put, the program set the arm time on the counter, read and stored each time step in an array, sent that array to be analyzed and graphed, compared that graph with the hypothesized graph, and ran statistics comparing each graph. For the finalized results we sent the data from Labview to

Microsoft Excel to be graphed, the content was the same but in Excel it is presented in a neater fashion.

Fig. 3 Labview Program

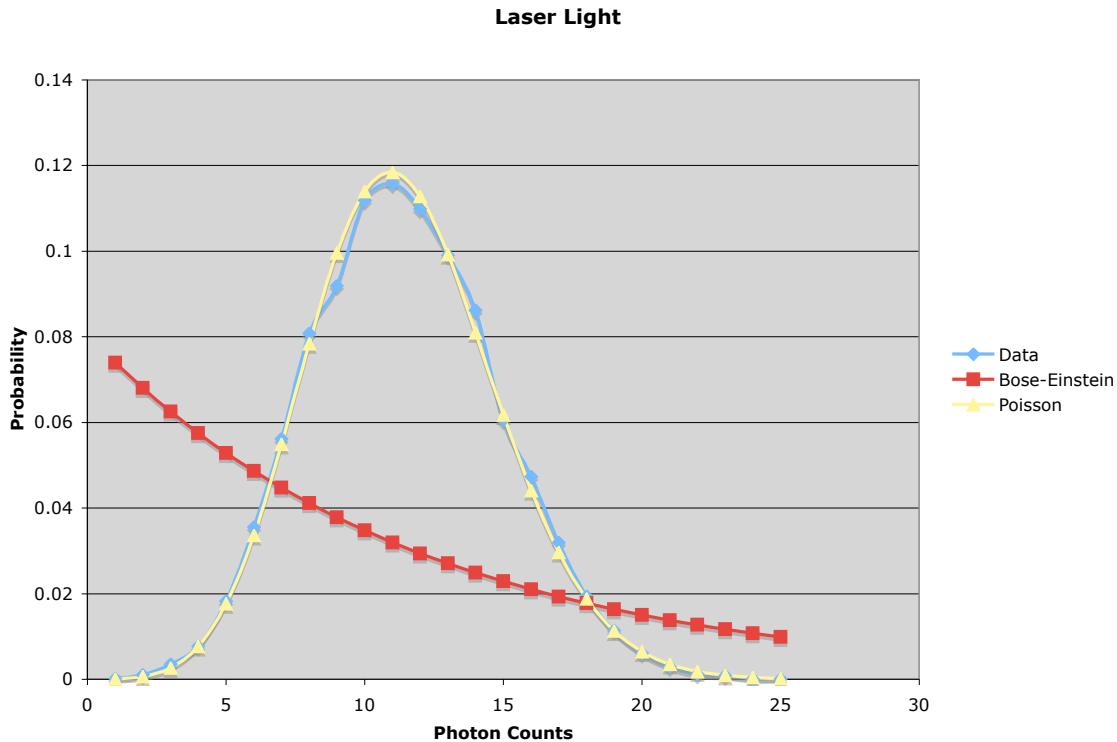


IV. Data

The data gathered (figs. 4 and 5) are represented as graphs, each with a line of points representing the data, the Poisson fit, and the Bose-Einstein fit. The y-axis is always the probability of an event occurring while the x-axis is the corresponding event, photon count per time step. The range of the x-axis was always the spread between and including the minimum and maximum counts. To analyze these graphs we looked at two sets of statistics; the r^2 correlation to a particular fit and the variance value to the fits hypothesized values (shown in the background section,) the latter of which we called the CL or confidence level. The closer the r^2 coefficient is to 1 the closer the data matches a particular fit and the closer the CL is to 100 the better the variance fit is. The r^2 value was found via statistics comparing each data point to both the Poisson and Bose-Einstein fit to see which it emulates more. The CL is calculated by dividing the fits variance by the data's variance then multiplying by 100.

There are two sets of data, one corresponding to data that matches a Poisson fit and one set of data that matches a Bose-Einstein fit. Each set of data consists of laser data with a small range and a large range; the Poisson data has a couple more graphs also but those will be explained later. The reason for the need of both small and big ranges is for accuracy in interpreting the data. With the small range the data tends to follow the fit more closely, however due to the similarity of the graphs and the r-squared values between both fits it can't be 100 percent determined that it only follows that particular fit. This is more prevalent with the pseudo-thermal graph and is what led to the need of the large range. The large range data requires more data points for accuracy and has a slightly smaller r^2 value but it clearly distinguishes one fit from the other.

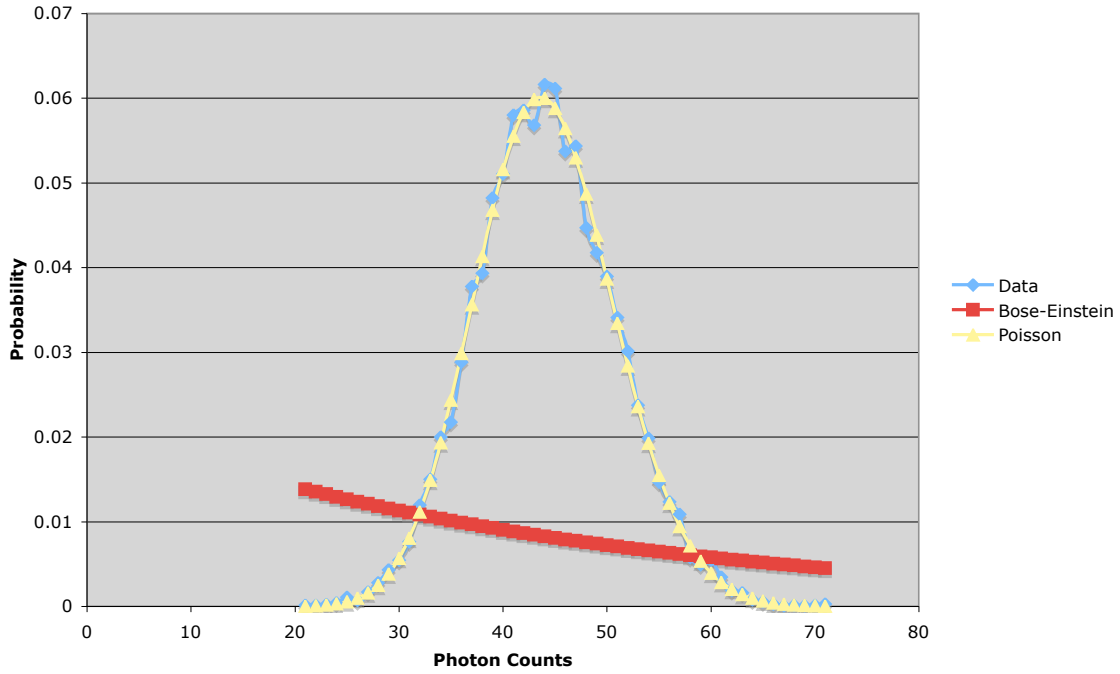
Fig. 4 Poisson Data



$$r^2: \text{Poisson} = 0.997 \quad \text{Bose-Einstein} = 0.124$$

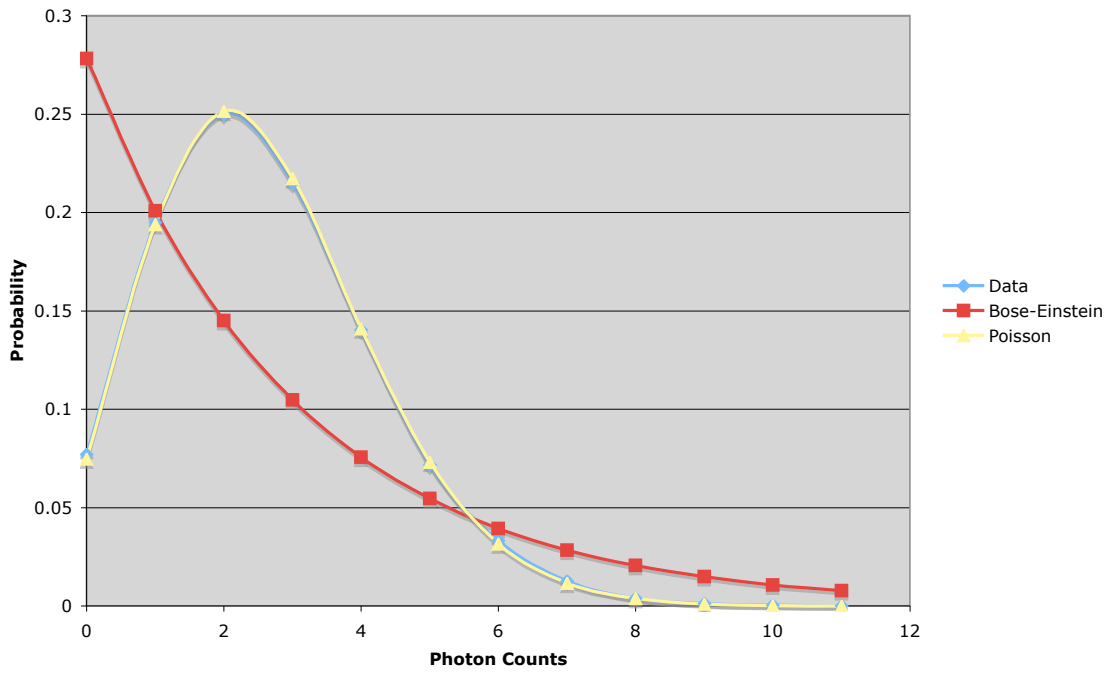
$$\text{CL: Poisson} = 99.0148 \quad \text{Bose-Einstein} = 1230.994$$

Laser Light

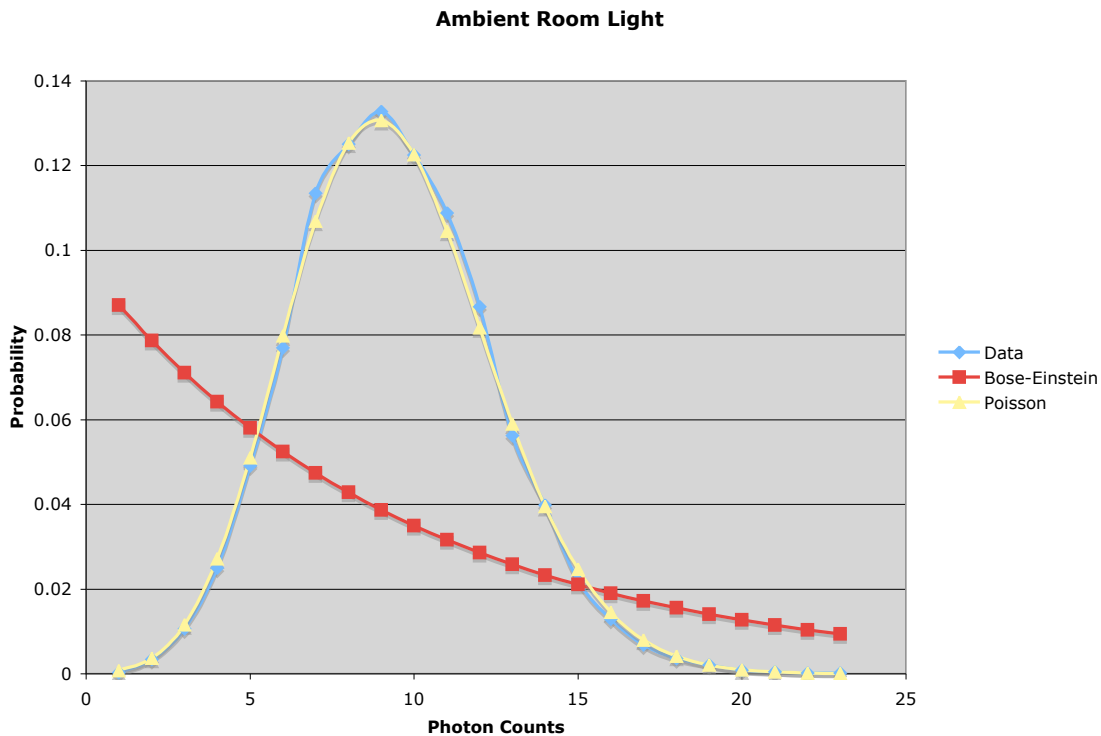


r^2 : Poisson = 0.996 Bose-Einstein = -0.247
CL: Poisson = 98.608 Bose-Einstein = 4449.114

True Thermal



r^2 : Poisson = 0.99987 Bose-Einstein = 0.808
CL: Poisson = 97.839 Bose-Einstein = 351.549



$$r^2: \text{Poisson} = 0.998 \quad \text{Bose-Einstein} = 0.270$$

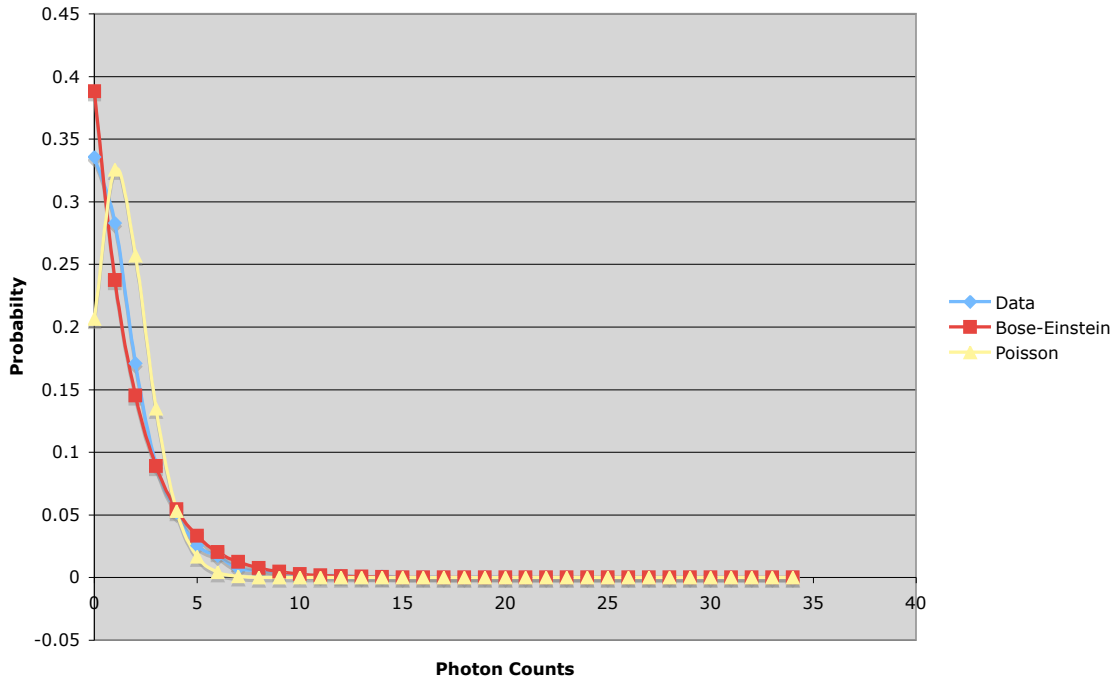
$$\text{CL: Poisson} = 104.742 \quad \text{Bose-Einstein} = 1087.487$$

These four graphs all represent data that matches a Poisson fit. The first two graphs are the two necessary ranges needed to prove the fit while the next two show that the Poisson distribution is the default distribution for most photon statistics. Each is the product of a light source firing into the PMT without destructive interference.

The first of the latter two graphs is data points from counting a true thermal source. This source was a general electric 100W incandescent light bulb, taken to show the coherence time dilemma. As mentioned early the thermal source follows the Bose-Einstein distribution, with zero counts as the most probable count. What this data shows is even a thermal source, when taken outside the coherence time, defaults to a Poisson distribution. The last graph also shows that the Poisson is the default distribution because it is regular light coming from a halogen bulb. This shows that the Poisson fit wasn't the hard data to gather; that honor falls to the Bose-Einstein fit.

Fig. 5 Pseudo-thermal data

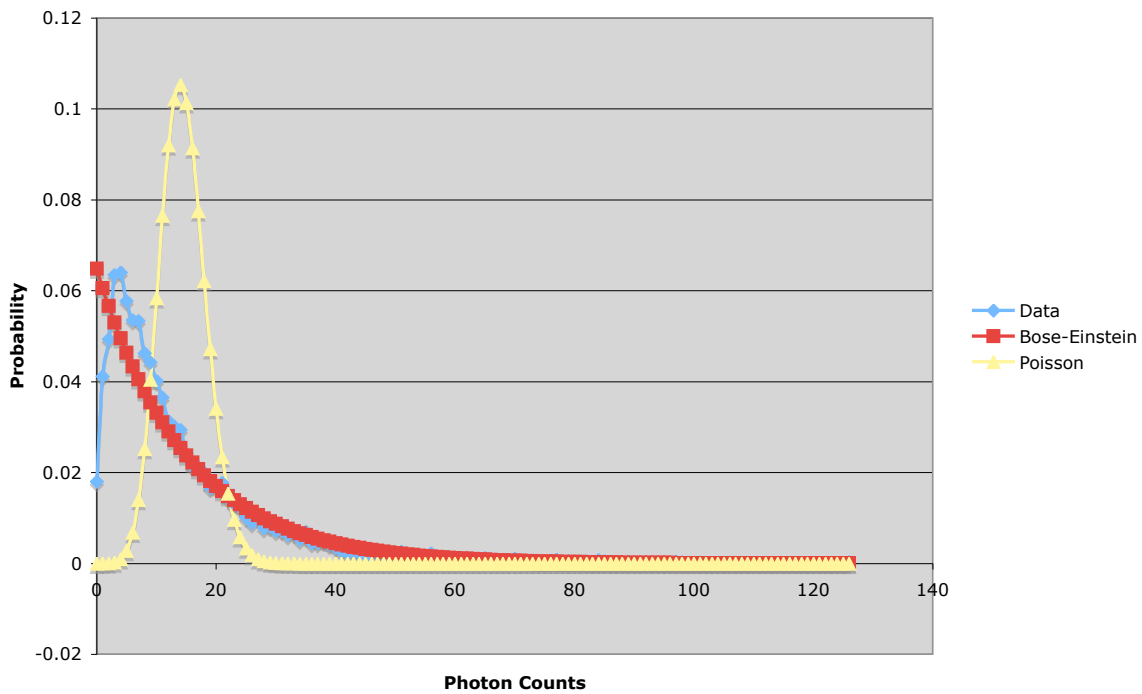
Slow Spin at 0.001s Arm Time



r^2 : Poisson = 0.9567 Bose-Einstein = 0.993

CL: Poisson = 39.745 Bose-Einstein = 102.423

126 Range 0.5 V spin



r^2 : Poisson = -0.0862 Bose-Einstein = 0.9846

CL : Poisson = 6.452 Bose-Einstein = 99.552

These graphs correspond to the pseudo-thermal data that follow the Bose-Einstein distribution. These results were much more difficult to achieve because of all the adjustments that could be made to the setup. These adjustments included: grit of the ground glass, rotation speed of the ground glass, arm time on the counter, angle of the polarizers, and apparatus distances. Granite, the Poisson distribution setup also required an adjustment in polarizer angle, arm time, and distance, but because it is the default distribution no matter what these were set to the pattern was almost guaranteed.

The best setup that emulated the effects of thermal light consisted of the largest grit glass, the smallest arm time, and the slowest spin. The glass used was 120 grit, the arm time was 1ms, and the spin was between 3 and 5 volts. Both setups had the PMT 30 cm away from the ground glass and the ground glass 30 cm away from the laser source. The data from the first graph was collected while the polarizers were set 30 degrees apart while the data on the second was absent polarizers.

The smallest arm time is essential because it needs to be sufficiently lower than the coherence time to get Bose-Einstein data. In the second graph the data points dont fit exactly the Bose-Einstein fit because of the fact that although the laser has a much larger coherence time than actual thermal light 1ms isnt a perfectly sufficient arm time. This means that the data will have a smaller count in the zero and low count sections because the number of time steps that are caught in between two sets of coherence time steps. When the arm time was increased from 1ms to 10 ms a mixture of both Poisson and Bose-Einstein distributions was found in the data. This is because the arm time was still smaller than the coherence time so some of the data followed the Bose-Einstein distribution, but at the same time the percentage of time steps that were a mixture of two different coherence time steps increased allowing for more of the default distribution. When only the arm time changed the r-squared data was as follows: 0.658 for Poisson and 0.400 for Bose-Einstein.

Increasing the spin or having no spin caused similar affects to the data as increasing the arm time. As the spin increased the light shinning through the ground glass acted more Poisson and less Bose-Einstein. This is because the faster the spin of the glass the less amount of time the destructive interference, caused by the speckle, had to effect that time steps data; thus decreasing the amount of zero counts leading it away from pseudo-thermal properties. No spin means the data couldnt be manipulated by the changing speckle property of the ground glass allowing it to stay constant at a particular speckle. This led to data similar to shinning the laser at the PMT with no ground glass.

The r^2 data for fast spin at 1.7 volts and no spin were as follows: 0.666 for Poisson and 0.112 for Bose-Einstein, and 0.832 for Poisson and -0.0303 for Bose-Einstein. This means that the fast spin data was a mixture of both distributions, similar to that of a larger arm time, while no spin acted more purely Poisson. Finding the correct range to get mostly the Bose-Einstein distribution required trial and error.

V. Conclusion

Before concluding the results it is relevant to mention that these counts arent the exact number of photon leaving the source in a particular time step. Firstly, only a portion of the light is counted, that portion that enters through the 10-20 μ m pinhole. Secondly, whenever light travels through optics a certain percentage is block or dispersed, such as in the ground glass. Thirdly the polarizers block a good percentage of the light, following the formula $I = I_0 \cos^2(\theta)$ where θ is the angle between the polarizers and I_0 is the initial intensity. Fourthly the PMT itself has a specific efficiency in detecting a photon (fig. 2 b). Adding all of these together we find that the count we record is but a fraction of the actual intensity, one that has the same distribution and statistical properties.

From our data we can conclude that Poisson is the default distribution for photon counts. This

distribution not only occurs when a light source is monochromatic but also when the time step or spin is too large for a thermal source. It is this coherence time property of light that determines the necessary time step needed for a specific distribution such as the Bose-Einstein distribution. Our second set of data shows that Bose-Einstein is the distribution pattern for thermal light provided this time step is well below the coherence time.

Our conclusion was found via statistics by comparing r^2 and variance of a sample of data compared to a set distribution. By combining both small and large range data sets one can determine with certainty the above results. The Bose-Einstein result is the tougher of the two to achieve. The short range Bose-Einstein distribution greatly resembles the Poisson distribution so the data is not conclusive. The long-range data is more conclusive however the first couple counts are off, due to the time step of the counter being a little too large for the coherence time of the laser. With all this in mind the data is conclusive that a monochromatic laser source follows a Poisson distribution while a laser pseudo-thermal source mimics the Bose-Einstein distribution held by a thermal source.

VI. Literature Cited

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