

# The Physics of Soccer: Bounce and Ball Trajectory

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The total trajectory of a soccer ball post stationary kick was simulated using MATLAB. The five forces on the ball are: gravity, drag, normal, friction (both rolling and sliding), and magnus. The coefficients for each force were determined using literature and experimental data. A Runge-Kutta fourth order numerical approximation was used to evaluate the differential equation found from Newtons second law. Also, translational and rotational velocity shifts due to bounce were theoretically examined, however sufficient data were not recorded to verify.

## I. Background Information

All around the world sports play a large impact on the community. All communities cheer on their respective teams with zeal and enthusiasm, many times culminating in spending vast amounts of money. The popularity of sports and their impact on economies make them an important area of scientific research. Any team that can get an edge over another has a better chance of winning, leading to happy fans and more money. One way to gain this edge is to analyze the given sport, and determine how to produce the desired effects, i.e. scoring goals in soccer.

There are many different sports, each with its own fan base, none of which are more enjoyed than soccer. Much of the scientific research done on it has focused on the analysis of the game, mainly the physics behind a soccer kick. There are two main branches for the scientific study of the soccer kick: analysis of the ball while it is in motion and analysis of the foot-ball interaction<sup>1</sup>: the former being more highly studied than the latter. Analysis of the ball in flight falls within the field of computational fluid dynamics and takes into consideration the complex effects of airflow and drag forces on the ball, used to determine the trajectory of the ball once it is kicked. Substantial work has been done, leading to simulation packages and redesigning of the soccer ball surface.

Our work continues on earlier research, aimed to combine numerous ideas into one cohesive and accurate trajectory program. The ultimate goal for the program is to map the entire trajectory of the soccer ball using only initial conditions and comparing the theoretical flight to experimental data. Accuracy of the program relies on many separate areas of research, such as: programming, flight (determining coefficients), spin decay, friction, and velocity change upon bounce.

## II. Flight Theory

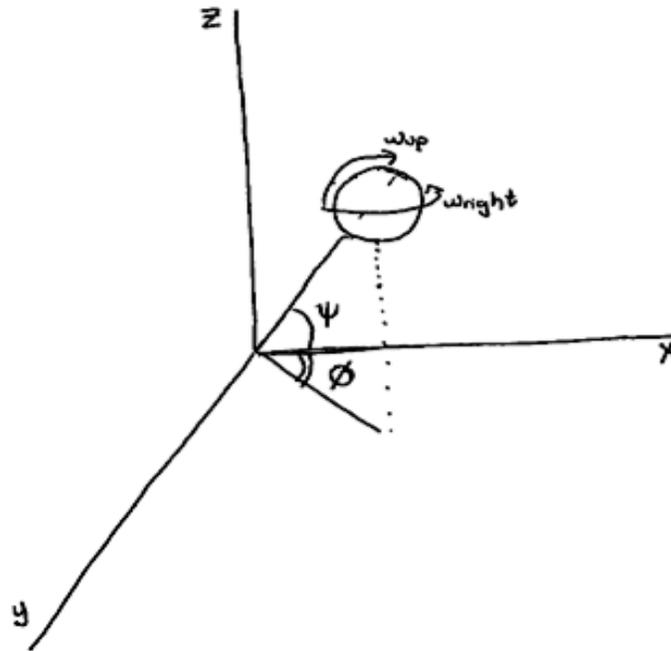
To determine the trajectory of a soccer ball one must find the initial conditions of the ball and the forces on the ball as it is in flight. Once these are known the balls trajectory from kick to just prior to it contacting the ground can be found. This portion of the flight has been investigated heavily for all types of projectiles; e.g. golf balls, bullets, and soccer balls.

Prior to inputting the initial conditions, we set our coordinate system, figure 1. Cartesian coordinates were used with z-direction corresponding to the height of the ball off of the ground (assuming the ground is flat), x-direction corresponding to the most profound direction of the kick (from the ball to the target), and the y-direction corresponding to the distance off the x-axis the ball travels with respect to the ground. The starting location of the ball was usually considered the origin, however this could change depending on the type of kick.

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<sup>1</sup>Kellis, Eleftherios, and Athanasios Katis. "Biomechanical Characteristics and Determination of Instep Soccer Kick." *Journal of Sport Science and Medicine* 6.2 (2007); 154+. Academic One File. Web. 16 Mar. 2010.

Fig. 1 Coordinate System



The initial conditions needed for our program include: initial ball speed  $v$ , initial ball position  $x$ ,  $y$  and  $z$ , initial back/topspin  $\omega_{up}$ , initial sidespin  $\omega_{right}$  (there is a third possible spin degree of freedom however it is impossible to impart velocity and this spin on the ball from a kicking position), and initial angles. The two angles needed are  $\psi$ , the angle between the  $z$ -axis and  $x$ - $y$  plane, and  $\phi$ , the angles between the  $x$ -axis and  $y$ -axis. Data from a high-speed video camera could be used to obtain the initial conditions for a ball in flight.

Initial conditions can vary substantially depending on the foot-ball interaction. A player intending to curve the ball into a target area, such as in a corner kick situation will put much more spin on the ball as opposed to a driven ball. Ball speed can also vary depending on the strength of the kick, usually determined by the distance of the intended pass or shot. An example<sup>2</sup> of a successful corner kick into the target area has the initial conditions of:  $24.6 \text{ m/s} < v < 29.1 \text{ m/s}$ ,  $3 \text{ rev/s} < \omega_{right} < 9 \text{ rev/s}$ ,  $24^\circ < \psi < 28^\circ$ , and  $-2^\circ < \phi < 2^\circ$ .

Studies<sup>3</sup> have also been conducted on maximum amount of possible spin imparted on a soccer ball from a kick. When the ball is contacted furthest from the center, a distance equal to the radius is the maximum possible distance, the ball will have the smallest amount of linear velocity with a maximum amount of angular velocity. Taking a foot impact velocity of  $25 \text{ m/s}$ , the maximum possible angular velocity was found to be  $16.2 \text{ rev/s}$  (taking into account this will not be the same for every soccer ball). Therefore, for a foot impact speed of  $25 \text{ m/s}$ , there are many possible initial conditions varying with angular velocity in both the topspin and sidespin directions, and both angles, not to mention linear velocity can change as well.

The next step is to determine the forces of the ball in flight: gravity, drag, and magnus. The magnus force acts on the ball according to spin, therefore can be broken into two components, one

<sup>2</sup>Cook, Brandon and John Goff, "Parameter space for successful soccer kicks." *European Journal of Physics* 27 (2006) pp. 865-874; Academic One File, Web.

<sup>3</sup>Carre, M and S. Goodwill and S. Haake, "Understanding the effect of seams on the aerodynamics of an association football." *Journal of Mechanical Engineering Science* (2005) vol 219 pp. 657-666.

for topspin  $F_L$ , which is responsible for lift, and one for sidespin  $F_S$ , which is responsible for curve. Determining the magnitude of the force in each direction is equivalent to determining the amount of topspin and sidespin a ball has instead of total spin. The equations used for each force are<sup>4</sup>:

$$\vec{F}_D = \frac{1}{2}\rho Av^2 C_D(-\hat{v}) \quad (1)$$

$$\vec{F}_L = \frac{1}{2}\rho Av^2 C_L(\hat{l}) \quad (2)$$

$$\vec{F}_S = \frac{1}{2}\rho Av^2 C_S(\hat{l}x\hat{v}) \quad (3)$$

$$\vec{F}_G = -mg(\hat{z}) \quad (4)$$

where  $\rho$  is air density,  $A$  is cross-sectional area,  $m$  is ball mass, and  $v$  is velocity.

Each force equation has a coefficient depending on spin and velocity. If there is no spin then only gravity and drag act on the ball, and drag becomes an equation of purely linear velocity. However with spin, a new variable is defined,

$$Sp = \frac{r\omega}{v} \quad (5)$$

where  $r$  is the radius of the ball,  $\omega$  is angular speed, and  $v$  is linear velocity. The coefficient of drag, lift, and spin are determined from fitting these equations with data<sup>5</sup>:

$$C_D = \frac{a + b}{1 + \exp[(v - v_c)/v_s]} \quad (6)$$

for  $Sp < 0.3$  and

$$C_D = cSp^d \quad (7)$$

for all other  $Sp$ .  $a = 0.155$ ,  $b = 0.346$ ,  $c = 0.4127$ ,  $d = 0.3056$ ,  $v_c = 12.19$  m/s,  $v_s = 1.309$  m/s, and  $v$  is ball velocity.

$$C_L = C_S = 1.14Sp \quad (8)$$

for  $Sp < 0.280$ , and  $0.320$  otherwise. This was determined via linear fit from literature data. The differences between the two coefficient values are the direction and the magnitude of  $Sp$ . It is important to remember that these equations are subject to change depending on the make and type of the target being used, in our case an addidas jabulani soccer ball. After the force equations are determined, Newtons second law can be solved, and acceleration can be found.

$$\vec{a} = \beta v^2[-C_D(-\hat{v}) + C_L(\hat{l}) + C_S(\hat{l}x\hat{v})] + g(\hat{z}) \quad (9)$$

where  $\beta = \rho A/2m$  approximately equalling  $0.0530$  1/m, a relative constant.

<sup>4</sup>Carre, Matt and John Goff. "Trajectory analysis of a soccer ball." *American Journal of Physics* (2009) 77 (11), pp. 1020-1027.

<sup>5</sup>Carre, Matt and John Goff. "Trajectory analysis of a soccer ball." *American Journal of Physics* (2009) 77 (11), pp. 1020-1027.

Breaking the acceleration up into cartesian components yields the following equations:

$$a_x = \beta v^2[-C_D(\sin(\phi)\cos(\psi)) + C_L(-\cos(\phi)\cos(\psi)) + C_S(-\sin(\phi))] \quad (10)$$

$$a_y = \beta v^2[-C_D(\sin(\phi)\sin(\psi)) + C_L(-\cos(\phi)\sin(\psi)) + C_S(\cos(\psi))] \quad (11)$$

$$a_z = \beta v^2[-C_D(\cos(\phi)) + C_L(\sin(\phi))] + g \quad (12)$$

The products, second-order coupled nonlinear differential equations, must be solved numerically<sup>6</sup>, we used the Runge-Kutta fourth order method. A copy of the program can be seen in Appendix A.

There is one more force to account for before the trajectory can be mapped, the force that causes spin to decay over time. This was not included in the initial trajectory analysis because little research has been done on this force pertaining to soccer balls, however golf balls have been extensively examined. Because this force effects spin rate, which in turn affects both the Magnus and drag forces, it is important to look at and add into the program. It is suggested that due to its similar surface structure, the spin decay of a soccer ball should emulate that of a golf ball for turbulent flow, speeds greater than 10.3 m/s, where most soccer ball speeds will be<sup>7</sup>.

In some cases, the force responsible for spin decay is known as the moment force, having the equation:

$$T = C_m \frac{1}{2} \rho v^2 A d \quad (13)$$

The equation similar to other aerodynamic equations where  $d$  is the ball's diameter. The coefficient of moment,  $C_m$  can then be experimentally determined, just as the other coefficients in aerodynamics are. Treating this equation separate from the other force equations and using Newtons second law yields the equation for spin decay as<sup>8</sup>

$$\omega(t) = \omega_0 e^{\frac{-cv}{r}t} \quad (14)$$

where  $\omega_0$  is the initial spin,  $c$  is an experimental constant,  $r$  is the ball radius, and  $t$  is total elapsed time.

Using the initial conditions and the acceleration, the position of the ball with respect to time can be determined; thus equating to the flight trajectory. However this is not the end of the total trajectory or the physics needed to explain it. Once the ball impacts the ground, bounces, a new force acts on it causing "new" initial conditions. These conditions are then plugged into the flight equation again and used until the next bounce. This continues until the velocity of the ball approaches zero, finally terminating the program and allowing for complete trajectory mapping.

### III. Bounce Theory

A bouncing ball has been looked at for other sports, especially tennis, but not much research has involved soccer. It is complex due to the collision between the ball and the surface. The bounce

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<sup>6</sup>Carre, Matt and John Goff. "Trajectory analysis of a soccer ball." *American Journal of Physics* (2009) 77 (11), pp. 1020-1027.

<sup>7</sup>James, David and Steve Haake. "The spin decay of sports balls in flight." *The Engineering of Sport 7 vol. 2*; pp. 165-172.

<sup>8</sup>Tavares G. K. Shannon, and T. Melvin. Golf ball spin decay model based on radar measurements. *Science and Golf III: proceedings of the 1998 World Scientific Congress of Golf*; chapter 58. 1998.

causes the ball to deform, changing its shape dramatically. Assuming uniform deformation, the force and time of the bounce can be averaged.

Although it is convenient to say a bounce is instantaneous, it is far from the truth. A ball takes time to bounce, time for it to deform to max deformation, then form back into a ball. An average time for the bounce can be calculated using geometry and assuming the ball is a perfect sphere<sup>9</sup>.

$$t_b = \pi \sqrt{\frac{m}{cp}} \quad (15)$$

where  $m$  is the ball mass,  $c$  is the ball circumference, and  $p$  is the pressure inside the ball.

Energy is transferred during the deformation in the bounce. There is a loss of energy in the translational directions, but rotational energy can either be gained or loss depending on the conditions of the bounce. This is because friction adds a torque to the ball, and depending on initial spin the torque could add to the total spin or subtract from it. In addition to friction, there is the normal force acting on the ball, which pushes the ball in the positive z-direction.

There are two types of friction applied to the ball, rolling and sliding. Wesson makes the case that on a bounce a ball will either undergo one or the other, dependant on the angle and spin of approach. Wesson states the following inequality<sup>10</sup>

$$\mu_r(1 + e)v_0 > \frac{2}{5}(u_0 - \omega_0 r) \quad (16)$$

where if true the ball rolls on bounce, else it slides.  $\mu_r$  is the coefficient of rolling friction,  $e$  is the coefficient of restitution,  $v_0$  is the pre-bounce ball velocity in the z-direction,  $u_0$  is the pre-bounce ball velocity in the x-y plane, and  $r$  is the ball radius.

The coefficient of friction in both cases can be determined experimentally, using a high speed video camera. A ballpark figure of  $\mu_r = 0.07$  seems accurate for the coefficient of rolling friction between an addidas jabulani soccer ball and a grass field however enough data has not been collected to verify. The data that was collected was found by rolling a soccer ball on a grass field. The beginning of the roll was filmed by a Casio Ex-F1 1200 fps camera and the initial velocity was determined. From here the total distance traveled was measured. Using the following kinetics equation:

$$v^2 = v_0^2 - 2a\Delta x \quad (17)$$

where  $a$  is acceleration. Solving Newton's second law and assuming only friction and gravity are acting on the ball,  $a = -\mu_r g$ . Then, solving for  $a$  in equation 17 gives an equation for  $\mu_r$ . It is important to note that different balls and different surfaces will have different coefficients of friction.

Wesson continues to theoretically show the change in rotational and translational upon a bounce, however because his work is only in two dimensions, dealing only with spin in the direction of the ball's velocity, it is not entirely accurate. We propose two possible introductory solutions to solving for the balls new "input conditions" using torques and energies, however sufficient data has not been collected to verify.

When the ball contacts the ground friction adds a torque; if it rolls it pulls the bottom of the ball back adding topspin. The strength of the force of friction is related to the frictional coefficient between the two contact surfaces. The force of the bounce can be written as:

<sup>9</sup>Wesson, John. "Mathematics." *The Science of Soccer* p. 146. Taylor and Francis Group, New York, NY, 2002.

<sup>10</sup>Wesson, John. "Mathematics." *The Science of Soccer* p. 154. Taylor and Francis Group, New York, NY, 2002.

$$F_b = \mu mg \quad (18)$$

where  $\mu$  is either sliding or rolling,  $m$  is the ball mass, and  $g$  is the gravitational constant.

Since the bounce occurs at a maximum distance away, the radius, from the center it adds the maximum possible spin from that force. Solving Newtons second law yields the following equation for topspin after the bounce (Appendix B):

$$\omega_{up} = \omega_{up0} + \frac{F_b}{C_r m r} t_b \quad (19)$$

where  $\omega_{up0}$  is the pre-bounce top/backspin speed and  $C_r$  is the moment of inertia coefficient for the soccer ball.

This analysis is useful to determine the change in topspin/backspin of the ball from the bounce. Sidespin and linear velocity on the other hand are not involved in this theory, and determining their values after the bounce would involve other theory, such as energy.

Another way we propose to analyze trajectory change upon bounce is using energy. Examining how much energy is conserved after the bounce would be a valuable tool in determining how the bounce effects the trajectory. Assuming normal, gravity, and frictional forces are the only forces on the ball during the bounce yields the following equation for the post bounce velocity in the x and y directions,

$$v_i = -\mu g t_b + v_{i0} \quad (20)$$

while the post bounce velocity in the z-direction is simply the coefficient of restitution multiplied by the pre-bounce z-velocity.

$$v_z = e v_{z0} \quad (21)$$

$\mu$  is the specific coefficient of friction,  $i$  is either x or y,  $v_{i0}$  is the pre-bounce velocity in the  $i$  direction, and  $v_z$  is the pre-bounce velocity in the z-direction.

Next, looking at the change of topspin via conservation of energy and solving for new top/back spin yields (Appendix C):

$$\omega_{up} = \sqrt{\omega_{up0}^2 + \frac{1}{C_r r^2} [v_{x0} \mu g t_b + v_{y0} \mu g t_b + v_{z0}^2 (1 - e)]} \quad (22)$$

Comparing this topspin change with that predicted by torque theory should yield similar results. There is one set back however, translational and rotational energy are not conserved. To correct this, we propose running experiments to find a new coefficient, dubbed the coefficient of conservation.

A proposed experiment would be to record the total translational and rotational energy of the soccer ball pre-bounce and compare it to the energy post-bounce. Due to loss of energy via friction, ball deformation, and elastic collision, the total translational and rotational energy post-bounce will be less than that of pre-bounce. Dividing the new energy value by the old value will yield a ratio, similar to the coefficient of restitution: the coefficient of conservation.

Multiplying this coefficient by the linear and angular velocity equations from the bounce should yield a theoretical answer that is in concordance with experimentation. This coefficient will change depending on the properties of the bounce, ball material, surface, angle of approach, spin, and

other variables so experimentation to determine which conservation value will be accurate for the particular kick of interest would be necessary.

To find these different coefficients of conservation, one would have to record the energy change for different bounces: large angle, small angle, large spin, small spin, etc. and come up with a basis where any linear combination would span all possible coefficient of conservation values. One such experimental example would be to determine the coefficient for each extreme separately, then determine how the kick of interest is composed of each component.

Two high-speed cameras, such as the Casio Ex-F1 (shooting at 1200 fps) are needed to successfully attain accurate data for both angular velocity and linear velocity pre and post bounce. The best two locations for filming the bounce for the cameras are directly behind the path of the ball, such as behind the kicker, and facing the ball at the bounce area. These views are orthogonal, gathering information on the entire moving ball. The camera behind the ball will capture information on sidespin, topspin, y-velocity, and z-velocity, while the camera positioned at the bounce will capture information on sidespin, topspin, x-velocity, and z-velocity. An important note is that when filming, a camera should be moved as far away from the target as applicable with the optical zoom engaged in order to limit perspective error.

Tracking the linear velocity is simple, track a known position on the ball with respect to time: such as the very bottom or center of the ball. Tracking the angular velocity is more complicated, but a method using one camera is suggested here. To track angular velocity, one must track a point on the ball as it moves across it. This point is moving due to the spin on the ball as well as the linear velocity, therefore the known linear velocity, gathered from tracking another known point, must be subtracted from it. The value one is left with can be called the velocity of the point,

$$v_p = \frac{2r}{\Delta t} \quad (23)$$

where  $r$  is the ball radius and  $\Delta t$  is the elapsed time of the tacking.

Picking a point on the ball that traverses the approximate equator of the ball with respect to spin is important. When this is done one can estimate that the point travels across the diameter of the ball, dividing it by time to get the point velocity in m/s. There is error in tracking with this method, because the three-dimensional spherical shape of the ball. The assumption here is that the ball traverses the surface of the sphere at a constant velocity with respect to the two-dimensional view of the camera, while in reality the point travels across the ball faster when the point is in the middle of the ball. This is negated however by taking the average time from which the point travels across the surface.

From the velocity of the point one can solve for the change in time. Then, using the fact that when the point traverses a diameter of a sphere it travels  $\pi$  radians (for it has rotated half of a revolution) one can solve for angular velocity.

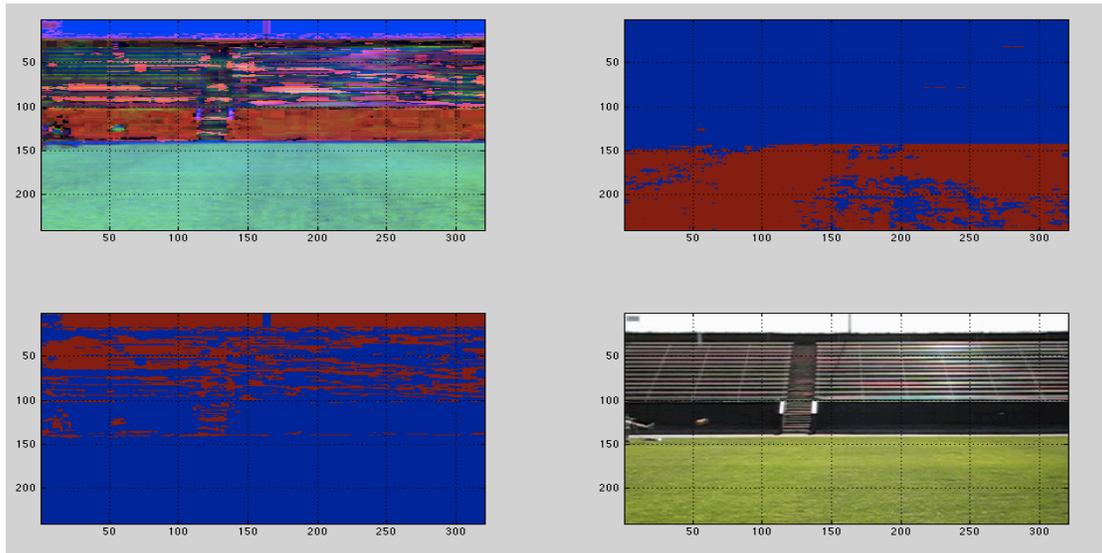
$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi v_p}{2r} \quad (24)$$

where  $\Delta\theta$  is the change in angle tracked.

Tracking the angular velocity is difficult to achieve across the duration of a flight, due to the necessity to zoom in to achieve a clear view of a point that can be tracked. This implies that only a small portion of the flight can have the angular velocity tracked, such as the first bounce. The linear velocity is simpler to track and therefore can be determined using a tracking program such as MATLAB, but due to the complexity of the angular velocity, tracking is best achieved by hand using a program such as Logger Pro.

Fig. 2 Tracking Program

The top two left picture represents the still-image, one movie frame, as a HSV image. The top right picture represents the targeted area specified by the wanted HSV values. The bottom right picture represents the still-image as and RGB image. The bottom left picture represents the targeted area specified by the wanted RGB values. Overlapping the top right and lower left pictures will give the wanted area, the ball, for the program to track.



A picture example of a tracking program written in MATLAB (Appendix D) can be seen in figure 2. This program can break a movie up into individual frames, then analyze each frame to locate the ball in its pixel array. The ball can be determined from the background of the image by setting an RGB and a HSV specific value for the color of the ball being used and overlapping. Then, finding the median pixel for the ‘ball location,’ yields the pixel/position of the ball with respect to time, the translational velocity. Again, two cameras would be needed to capture all three translational velocities and positions.

#### IV. Program

The trajectory program was written in MATLAB version R2007a (Appendix A). The user imputed the coefficient of restitution, coefficients of friction, ball pressure, ball radius, air density, cross-sectional area, mass, initial position, initial velocity, initial spin, initial angles, and the time step. From the initial conditions, the program calculated the aerodynamic coefficients using equations 6, 7 and 8.

After the coefficients are determined, the program enters a loop, provided the absolute velocity is greater than a desired value, but when less than the program exits because the ball is no longer moving. Then, using the time step and the acceleration equations: 10, 11, and 12, the program calculated the new positions and velocities via a Runge-Kutta fourth order numerical solutions method.

Next, the program calculated the new spin values based on the spin decay equation. Then, the program determines if the ball is in flight or bouncing by looking at the z-position value. If the ball is in flight the loop repeats, with the new position and velocities inserted into the Runge-Kutta equations, however, if the ball is bouncing, the program enters an if statement used to determine its properties. The time of the bounce is added to the total time and the inequality in equation 16 is used to determine if the ball is sliding or rolling.

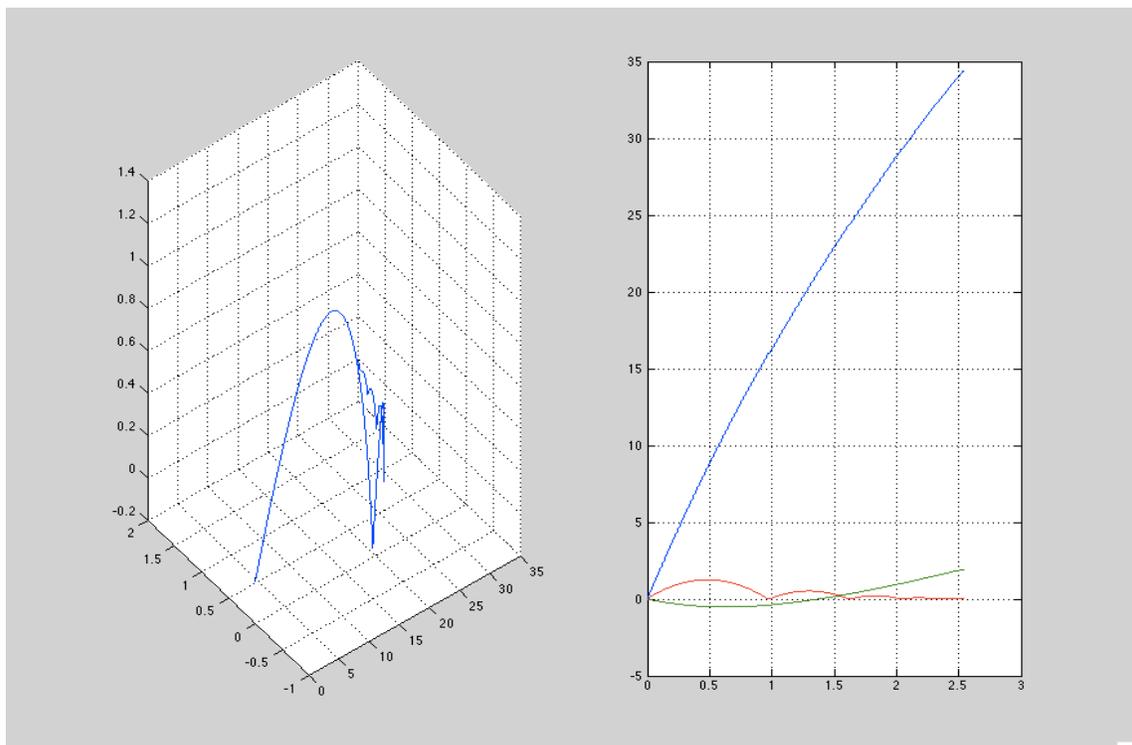
Depending on which the ball is doing upon the bounce different equations govern the change

in translational and rotational velocities. These equations still need to be improved however using either the energy or torque theory seems to be an adequate way to start. After the new values are determined, they are fed into the coefficient equations and then back into the flight loop as new “initial conditions.” This process repeats until the total velocity decreases below the chosen level. This seems to be an accurate method for determining the total trajectory of a soccer ball post kick because the ball is always bouncing, even if the pass is on the ground.

After the loop terminates the final position values are outputted into a matrix. These can be used to determine if the particular kick was successful, such as in Cook and Goffs paper; or used to match with data to see if the theory is accurate. Both are possible advances for the program but are currently still in progress. Finally, the program graphs the three positional arrays in two graphs, a three-dimensional graph representing the seen trajectory, and a two dimensional graph with the positional values on the y-axis vs. time on the x-axis. An output example can be seen in figure 3.

*Fig. 3 Program Output*

The left graph is the trajectory mapped in three dimension. The right graph is the trajectory components mapped in two dimensions against time. The initial conditions were:  $e = 0.65$ ,  $\mu_r = 0.07$ ,  $r = 0.11$  m,  $m = 0.424$  kg,  $x_i = 0$ ,  $y_i = 0$ ,  $z_i = 0$ ,  $v_i = 20$  m/s,  $\omega_{up_i} = 20$  rad/s,  $\omega_{right_i} = 5$  rad/s (spin pointing up),  $\phi = 15^\circ$ ,  $\psi = -5^\circ$  (initially traveling to the right), and  $\delta t = 0.001$  s.



Possible future work include: introducing the foot-ball interaction as the cause of the initial conditions, improve the tracking program to include automated tracking of spin, and matching real data with the theoretical trajectory to determine if the theory is accurate. Further research is also needed on the bounce including data to support the two proposed theoretical models. This information could be incorporated into the program to improve the overall accuracy of the predicted trajectory along with being used to determined the probability of a successful kick occurring.