2011 School of Science Summer Research Proposal

Project: Orthogonal Vector Coloring and Complement Graph

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An orthogonal vector coloring problem of a graph is an assignment of a vector to each vertex where the vectors assigned to the adjacent vertices must be orthogonal. Therefore, a question arises: what is the least number of dimensions of the vector space (which is denoted as the list-chromatic number of the graph) needed to assure the assignment to be complete? Followed by this question, what information can be gained about the list-chromatic number of the complement graph?

For the summer of 2011, my proposal for this research is to explore these two questions through analyzing possible vector space analogues of the list-chromatic number of a variety of graphs. The basis for this research originated from the research done by G. Haynes, C. Park, A. Schaeffer, J. Webster and L. H. Mitchell in 2008 in which they illustrated the orthogonal vector coloring by introducing two vector analogues of list coloring along with their chromatic numbers.

In our research, we expect to explore the relationship between the quantities relating to and centering at the vector chromatic number of a graph and basic information of the graph (e.g., the number and degree of the vertices) and those quantities and characteristics of its complement graph.

We will begin our research by adopting the coloring approach to investigate the vector coloring problem by assigning lists of vectors to each vertex so as to find a meaningful vector analogue for the list chromatic number. Because the presence or absence of an edge between two vertices dictates the list chromatic number of the graph and due to the unique linear algebraic properties an orthogonal vector coloring problem possesses, we will be able to use mathematical analysis software (e.g. *mathematica*) to help collect and process the data so as to develop and examine our hypothesis constructed based on the prior knowledge. We will then refine and derive a more clearly defined proposition and narrow down our subject by defining the terminology relates to the graph. We will test our proposition by examining the data collected and then generate proofs for the proposition.

After gaining sufficient information about the graph, we will attempt to deduce the connections between these "geometric dimensions" of a graph and the chromatic number of its complement graph. Applying the same procedure, we will begin our exploration by giving the vector equivalent of a well-known result that bounds the sum of the chromatic number of a graph and its complement. With the help of the mathematical analysis software, we will propose a reasonable hypothesis on the relation between the graph and its complement. We will then generalize the theories produced by G. Haynes, C. Park, A. Schaeffer, J. Webster and L. H. Mitchell concerning orthogonal vector coloring problem in which they characterize all graphs that have chromatic number two in each case and attempt to prove our hypothesis.

Acquiring this information of the orthogonal vector coloring problem could allow us to explore the quantum entanglement phenomena in quantum physics where the information encoded in a system of two photons can be possibly deciphered by our project. This application of our research will be further examined in the future.

Proposed Timeline

Week 1-2

Review literature pertaining to orthogonal vector coloring

Acquire the mathematics and physics concepts concerning the research

Week 3-5

Use mathematical analysis software to help collect and process the sample data

Develop the hypothesis based on the prior knowledge and the data collected

Week 6-8

Test the proposition by examining the data collected

Generate proofs for the proposition

Week 9-10

Report the findings and results and compose a paper with future recommendations

References

[1] G. Haynes, C. Park, A. Schaeffer, J. Webster and L. H. Mitchell, "Orthogonal Vector Coloring", 2008

[2] V. E. Alekseev and V. V. Lozin. Orthogonal Representations of Graphs. *Discrete Math.* 2001