TREATISE ON LIGHT

CONTAINING

THE EXPLANATION OF REFLECTION AND OF RE-
FRACTION AND ESPECIALLY OF THE RE-
MARKABLE REFRACTION WHICH
OCCURS IN ICELAND SPAR

BY

CHRISTIAAN HUYGENS

(Leyden, 1690)

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CHRISTIAAN HUYGENS

PREFACE

THIS treatise was written during my stay in Paris twelve years ago, and in the year 1678 was presented to the Royal Academy of Sciences, to which the king had been pleased to call me. Several of this body who are still living, especially those who have devoted themselves to the study of mathematics, will remember having been at the meeting at which I presented the paper; of these I recall only those distinguished gentlemen Messrs. Cassini, Römer, and De la Hire. Although since then I have corrected and changed several passages, the copies which I had made at that time will show that I have added nothing except some conjectures concerning the structure of Iceland spar and an additional remark concerning refraction in rock-crystal. I mention these details to show how long I have been thinking about these matters which I am only just now publishing, and not at all to detract from the merit of those who, without having seen what I have written, may have investigated similar subjects: as, indeed, happened in the case of two distinguished mathematicians, Newton and Leibnitz, regarding the question of the proper figure for a converging lens, one surface being given.

It may be asked why I have so long delayed the publication of this work. The reason is that I wrote it rather carelessly in French, expecting to translate it into Latin, and, in the meantime, to give the subject still further attention. Later I thought of publishing this volume together with another on dioptrics in which I discuss the theory of the telescope and the phenomena associated with it. But soon the subject was no longer new and was therefore less interesting. Accordingly I kept putting off the work from time to time, and now I do not know when I shall be able to finish it, for my time is largely occupied either by business or by some new investigation.

In view of these facts I have thought wise to publish this manuscript in its present state rather than to wait longer and run the risk of its being lost.

One finds in this subject a kind of demonstration which does not carry with it so high a degree of certainty as that employed in geometry; and which differs distinctly from the method employed by geometers in that they prove their propositions by well-established and incontrovertible principles, while here principles are tested by the inferences which are derivable from them. The nature of the subject permits of no other treatment. It is possible, however, in this way to establish a probability which is little short of certainty. This is the case when the consequences of the assumed principles are in perfect accord with the observed phenomena, and especially when these verifications are numerous; but above all when one employs the hypothesis to predict new phenomena and finds his expectations realized.

If in the following treatise all these evidences of probability are present, as, it seems to me, they are, the correctness of my conclusions will be confirmed; and, indeed, it is scarcely possible that these matters differ very widely from the picture which I have drawn of them. I venture to hope that those who enjoy finding out causes and who appreciate the wonders of light will be interested in these various speculations and in the new explanation of that remarkable property upon which the structure of the human eye depends and upon which are based those instruments which so powerfully aid the eye. I trust also there will be some who, from such beginnings, will push these investigations far in advance of what I have been able to do; for the subject is not one which is easily exhausted. This will be evident especially from those parts of the subject which I have indicated as too difficult for solution; and still more evident from those matters upon which I have not touched at all, such as the various kinds of luminous bodies and the whole question of color, which no one can yet boast of having explained.

Finally, there is much more to be learned by investigation concerning the nature of light than I have yet discovered; and I shall be greatly indebted to those who, in the future, shall furnish what is needed to complete my imperfect knowledge.

THE HAGUE, *8th of January*, 1690.

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CHAPTER I

ON THE RECTILINEAR PROPAGATION OF RAYS

DEMONSTRATIONS in optics, as in every science where geometry is applied to matter, are based upon experimental facts; as, for instance, that light travels in straight lines, that the angles of incidence and reflection are equal, and that rays of light are refracted according to the law of sines. For this last fact is now as widely known and as certainly known as either of the preceding.

Most writers upon optical subjects have been satisfied to assume these facts. But others, of a more investigating turn of mind, have tried to find the origin and the cause of these facts, considering them in themselves interesting natural phenomena. And although they have advanced some ingenious ideas, these are not such that the more intelligent readers do not still want further explanation in order to be thoroughly satisfied.

Accordingly, I here submit some considerations on this subject with the hope of elucidating, as best I may, this department of natural science, which not undeservedly has gained the reputation of being exceedingly difficult. I feel myself especially indebted to those who first began to make clear these deeply obscure matters, and to lead us to hope that they were capable of simple explanations.

But, on the other hand, I have been astonished to find these same writers accepting arguments which are far from evident as if they were conclusive and demonstrative. No one has yet given even a probable explanation of the fundamental and remarkable phenomena of light, *viz.*, why it travels in straight lines and how rays coming from an infinitude of different directions cross one another without disturbing one another.

I shall attempt, in this volume, to present in accordance with the principles of modern philosophy, some clearer and more probable reasons, first, for the rectilinear propagation of light, and, secondly, for its reflection when it meets other bodies. Later I shall explain the phenomenon of rays which are said to undergo refraction in passing through transparent bodies of different kinds. Here I shall treat also of refraction effects due to the varying density of the earth's atmosphere. Afterwards I shall examine the causes of that peculiar refraction occurring in a certain crystal which comes from Iceland. And lastly, I shall consider the different shapes required in transparent and in reflecting bodies to converge rays upon a single point or to deflect them in various ways. Here we shall see with what ease are determined, by our new theory, not only the ellipses, hyperbolas, and other curves which M. Descartes has so ingeniously devised for this purpose, but also the curve which one surface of a lens must have when the other surface is given, as spherical, plane, or of any figure whatever.

We cannot help believing that light consists in the motion of a certain material. For when we consider its production we find that here on the earth it is generally produced by fire and flame which, beyond doubt, contain bodies in a state of rapid motion, since they are able to dissolve and melt numerous other more solid bodies. And if we consider its effects, we see that when light is converged, as, for instance, by concave mirrors, it is able to produce combustion just as fire does; *i.e.*, it is able to tear bodies apart; a property that surely indicates motion, at least in the true philosophy where one believes all natural phenomena to be mechanical effects. And, in my opinion, we must admit this, or else give up all hope of ever understanding anything in physics.

Since, according to this philosophy, it is considered certain that the sensation of sight is caused only by the impulse of some form of matter upon the nerves at the base of the eye, we have here still another reason for thinking that light consists in a motion of the matter situated between us and the luminous body.

When we consider, further, the very great speed with which light is propagated in all directions, and the fact that when rays come from different directions, even those directly opposite, they cross without disturbing each other, it must be evident that we do not see luminous objects by means of matter translated from the object to us, as a shot or an arrow travels through the air. For certainly this would be in contradiction to the two properties of light which we have just mentioned, and especially to the latter. Light is then propagated in some other manner, an understanding of which we may obtain from our knowledge of the manner in which sound travels through the air.

We know that through the medium of the air, an invisible and impalpable body, sound is propagated in all directions, from the point where it is produced, by means of a motion which is communicated successively from one part of the air to another; and since this motion travels with the same speed in all directions, it must form spherical surfaces which continually enlarge until finally they strike our ear. Now there can be no doubt that light also comes from the luminous body to us by means of some motion impressed upon the matter which lies in the intervening space; for we have already seen that this cannot occur through the translation of matter from one point to the other.

If, in addition, light requires time for its passage—a point we shall presently consider—it will then follow that this motion is impressed upon the matter gradually, and hence is propagated, as that of sound, by surfaces and spherical waves. I call these *waves* because of their resemblance to those which are formed when one throws a pebble into water and which represent gradual propagation in circles, although produced by a different cause and confined to a plane surface.

As to the question of light requiring time for its propagation, let us consider first whether there is any experimental evidence to the contrary.

What we can do here on the earth with sources of light placed at great distances (although showing that light does not occupy a sensible time in passing over these distances) may be objected to on the ground that these distances are still too small, and that, therefore, we can conclude only that the propagation of light is exceedingly rapid. M. Descartes thought it instantaneous, and based his opinion upon much better evidence, furnished by the eclipse of the moon. Nevertheless, as I shall show, even this evidence is not conclusive. I shall state the matter in a manner slightly different from his in order that we may more easily arrive at all the consequences.

Let A be the position of the sun; BD a part of the orbit or annual path of the earth; ABC a straight line intersecting in C the orbit of the moon, which is represented by the circle CD.

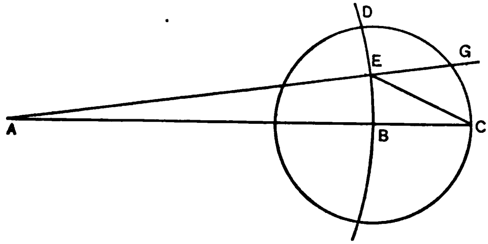

Fig. 1

If, now, light requires time — say one hour—to traverse the space between the earth and the moon, it follows that when the earth has reached the point B, its shadow, or the interruption of light, will not yet have reached the point C, and will not reach it until one hour later. Counting from the time when the earth occupies the position B, it will be one hour later that the moon arrives at the point C and is there obscured; but this eclipse or interruption of light will not be visible at the earth until the end of still another hour. Let us suppose that during these two hours the earth has moved to the position E. From this point the moon will appear to be eclipsed at C, a position which it occupied one hour before, while the sun will be seen at A. For I assume with Copernicus that the sun is fixed and, since light travels in straight lines, must always be seen in its true position. But it is a matter of universal observation, we are told, that the eclipsed moon appears in that part of the ecliptic directly opposite the sun; while according to our view its position ought to be behind this by the angle GEC, the supplement of the angle AEC. But this is contrary to the fact, for the angle GEC will be quite easily observed, amounting to about 33°. Now according to our computation, which will be found in the memoir on the causes of the phenomena of Saturn, the distance, BA, between the earth and the sun is about 12,000 times the diameter of the earth, and consequently 400 times the distance of the moon, which is 30 diameters. The angle ECB will, therefore, be almost 400 times as great as BAE, which is 5′, *viz.*, the angular distance traversed by the earth in its orbit during an interval of two hours. Thus the angle BCE amounts to almost 33°, and likewise the angle CEG, which is 5′ greater.

But it must be noted that in this argument the speed of light is assumed to be such that the time required for it to pass from here to the moon is one hour. If, however, we suppose that it requires only a minute of time, then evidently the angle CEG will amount to only 33′; and if it requires only ten seconds of time, this angle will amount to less than 6′. But so small a quantity is not easily observed in a lunar eclipse, and consequently it is not allowable to infer the instantaneous propagation of light.

It is somewhat unusual, we must confess, to assume a speed one hundred thousand times as great as that of sound, which, according to my observations, travels about 180 toises [1151 *feet*] in a second, or during a pulse-beat; but this supposition appears by no means impossible, for it is not a question of carrying a body with such speed, but of a motion passing successively from one point to another.

I do not therefore, in thinking of these matters, hesitate to suppose that the propagation of light occupies time, for on this view all the phenomena can be explained, while on the contrary view none of them can be explained. Indeed, it seems to me, and to many others also, that M. Descartes, whose object has been to discuss all physical subjects in a clear way, and who has certainly succeeded better than any one before him, has written nothing on light and its properties which is not either full of difficulty or even inconceivable.

But this idea which I have advanced only as a hypothesis has recently been almost established as a fact by the ingenious method of Römer, whose work I propose here to describe, expecting that he himself will later give a complete confirmation of this view.

His method, like the one we have just discussed, is astronomical. He proves not only that light requires time for its propagation, but shows also how much time it requires and that its speed must be at least six times greater than the estimate which I have just given.

For this demonstration, he uses the eclipses of the small planets which revolve about Jupiter, and which very often pass into its shadow. His reasoning is as follows: Let A denote the sun; BCDE, the annual orbit of the earth; F, Jupiter; and GN, the orbit of the innermost satellite, for this one, on account of its short period, is better adapted to this investigation than is either of the other three. Let G represent the point of the satellite's entrance into, and H the point of its emergence from, Jupiter's shadow.

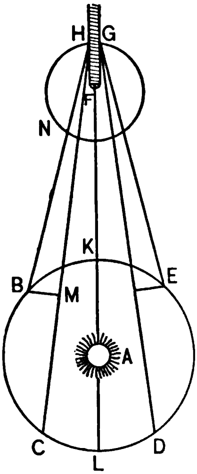

Fig. 2

Let us suppose that an emergence of this satellite has been observed while the earth occupies the position B, at some time before the last quarter. If the earth remained in this position, 42½ hours would elapse before the next emergence would occur. For this is the time required for the satellite to make one revolution in its orbit and return to opposition with the sun. If, for instance, the earth remained at the point B during 30 revolutions, then, after an interval of 30 times 42½ hours, the satellite would again be observed to emerge. But if meanwhile the earth has moved to a point C, more distant from Jupiter, it is evident that, provided light requires time for its propagation, the emergence of the little planet will be recorded later at C than it would have been at B. For it will be necessary to add to this interval, 30 times 42½ hours, the time occupied by light in passing over a distance MC, the difference of the distances CH and BH. In like manner, in the other quarter, while the earth travels from D to E, approaching Jupiter, the eclipses will occur earlier when the earth is at E than if it had remained at D.

Now by means of a large number of these eclipse observations, covering a period of ten years, it is shown that these inequalities are very considerable, amounting to as much as ten minutes or more; whence it is concluded that, for traversing the whole diameter of the earth's orbit KL, twice the distance from here to the sun, light requires about 22 minutes.

The motion of Jupiter in its orbit, while the earth passes from B to C or from D to E, has been taken into account in the computation, where it is also shown that these inequalities cannot be due either to an irregularity in the motion of the satellite or to its eccentricity.

If we consider the enormous size of this diameter, KL, which I have found to be about 24 thousand times that of the earth, we get some idea of the extraordinary speed of light.

Even if we suppose that KL were only 22 thousand diameters of the earth, a speed covering this distance in 22 minutes would be equivalent to the rate of one thousand diameters per minute, *i.e.*, $16\frac{2}{3}$ diameters a second (or a pulse-beat), which makes more than eleven hundred times one hundred thousand toises [212,222 *kilometres*], since one terrestrial diameter contains 2865 leagues, of which there are 25 to the degree, and since, according to the exact determination made by Mr. Picard in 1669 under orders from the king, each league contains 2282 toises.

But, as I have said above, sound travels at the rate of only 180 toises [350 *metres*] per second. Accordingly, the speed of light is more than 600,000 times as great as that of sound, which, however, is a very different thing from being instantaneous, the difference being exactly that between a finite quantity and infinity. The idea that luminous disturbances are handed on from point to point in a gradual manner being thus confirmed, it follows, as I have already said, that light is propagated by spherical waves, as is the case with sound.

But if they resemble each other in this respect, they differ in several others—*viz.*, in the original production of the motion which causes them, in the medium through which they travel, and in the manner in which they are transmitted in this medium.

Sound, we know, is produced by the rapid disturbance of some body (either as a whole or in part); this disturbance setting in motion the contiguous air. But luminous disturbances must arise at each point of the luminous object, else all the different parts of this object would not be visible. This fact will be more evident in what follows.

In my opinion, this motion of luminous bodies cannot be better explained than by supposing that those which are fluid, such as a flame, and apparently the sun and stars, are composed of particles that float about in a much more subtle medium, which sets them in rapid motion, causing them to strike against the still smaller particles of the surrounding ether. But in the case of luminous solids, such as red-hot metal or carbon, we may suppose this motion to be caused by the violent disturbance of the particles of the metal or of the wood, those which lie on the surface exciting the ether. Thus the motion which produces light must also be more sudden and more rapid than that which causes sound, since we do not observe that sonorous disturbances give rise to light any more than that the motion of the hand through the air gives rise to sound.

The question next arises as to the nature of the medium in which is propagated this motion produced by luminous bodies. I have called it ether; but it is evidently something different from the medium through which sound travels. For this latter is simply the air which we feel and breathe, and which, when removed from any region, leaves behind the luminiferous medium. This fact is shown by enclosing a sounding body in a glass vessel and removing the atmosphere by means of the air-pump which Mr. Boyle has devised, and with which he has performed so many beautiful experiments. But in trying this it is well to place the sounding body on cotton or feathers in such a way that it cannot communicate its vibrations either to the glass receiver or to the air-pump, a point which has hitherto been neglected. Then, when all the air has been removed, one hears no sound from the metal even when it is struck.

From this we infer not only that our atmosphere, which is unable to penetrate glass, is the medium through which sound travels, but also that it is different from that which carries luminous disturbances; for when the vessel is exhausted of air, light traverses it as freely as before.

This last point is demonstrated even more clearly by the celebrated experiment of Torricelli. That part of the glass tube which the mercury does not fill contains a high vacuum, but transmits light the same as when filled with air. This shows that there is within the tube some form of matter which is different from air, and which penetrates either glass or mercury, or both, although both the glass and the mercury are impervious to air. And if the same experiment is repeated, except that a little water be placed on top of the mercury, it becomes equally evident that the form of matter in question passes either through the glass or through the water or through both.

As to the different modes of transmission of sound and light, it is easy to understand what happens in the case of sound when one recalls that air can be compressed and reduced to a much smaller volume than it ordinarily occupies, and that just in proportion as its volume is diminished it tends to regain its original size. This property, taken in conjunction with its penetrability, which it retains in spite of compression, appears to show that it is composed of small particles which float about, in rapid motion, in an ether composed of still finer particles. Sound, then, is propagated by the effort of these air particles to escape when at any point in the path of the wave they are more compressed than at some other point.

But the enormous speed of light, together with its other properties, hardly allows us to believe that it is propagated in the same way. Accordingly, I propose to explain the manner in which I think it must occur. It will be necessary first, however, to describe that property of hard bodies in virtue of which they transmit motion from one to another.

If one takes a large number of spheres of equal size, made of any hard material, and arranges them in contact in a straight line, he will find that, on allowing a sphere of the same size to roll against one end of the line, the motion is transmitted in an instant to the other end of the line. The last sphere in the row flies off while the intermediate ones are apparently undisturbed; the sphere which originally produced the disturbance also remains at rest. Here we have a motion which is transmitted with high speed, which varies directly as the hardness of the spheres.

Nevertheless, it is certain that this motion is not instantaneous, but is gradual, requiring time. For if the motion, or, if you please, the tendency to motion, did not pass successively from one sphere to another, they would all be affected at the same instant, and would all move forward together. So far from this being the case, it is the last one only which leaves the row, and it acquires the speed of the sphere which gave the blow. Besides this experiment there are others which show that all bodies, even those which are considered hardest, such as tempered steel, glass, and agate, are really elastic, and bend to some extent whether they are made into rods, spheres, or bodies of any other shape; that is, they yield slightly at the point where they are struck, and immediately regain their original figure. For I have found that in allowing a glass or agate sphere to strike upon a large, thick, flat piece of the same material, whose surface has been dulled by the breath, the point of contact is marked by a circular disk which varies in size directly as the strength of the blow. This shows that during the encounter these materials yield and then fly back, a process which must require time.

Now to apply this kind of motion to the explanation of light, nothing prevents our imagining the particles of the ether as endowed with a hardness almost perfect and with an elasticity as great as we please. It is not necessary here to discuss the cause either of this hardness or of this elasticity, for such a consideration would lead us too far from the subject. I will, however, remark in passing that these ether particles, in spite of their small size, are in turn composed of parts, and that their elasticity consists in a very rapid motion of a subtle material which traverses them in all directions and compels them to assume a structure which offers an easy and open passage to this fluid. This accords with the theory of M. Descartes, except that I do not agree with him in assigning to the pores the form of hollow circular canals. So far from there being anything absurd or impossible in all this, it is quite credible that nature employs an infinite series of different-sized molecules, endowed with different velocities, to produce her marvellous effects.

But although we do not understand the cause of elasticity, we cannot fail to observe that most bodies possess this property: it is not unnatural, therefore, to suppose that it is a characteristic also of the small, invisible particles of the ether. If, indeed, one looks for some other mode of accounting for the gradual propagation of light, he will have difficulty in finding one better adapted than elasticity to explain the fact of uniform speed. And this appears to be necessary; for if the motion slowed up as it became distributed through a larger mass of matter, and receded farther from the source of light, then its high speed would be lost at great distances. But we suppose the elasticity to be a property of the ether so that its particles regain their shape with equal rapidity whether they are struck with a hard or a gentle blow; and thus the rate at which the light moves remains the same [*at all distances from the source*].

Nor is it necessary that the ether particles should be arranged in straight lines, as was the case with our row of spheres. The most irregular configuration, provided the particles are in contact with each other, will not prevent their transmitting the motion and handing it on to the regions in front. It is to be noted that we have here a law of motion which governs this kind of propagation, and which is verified by experiment, *viz.*, when a sphere such as A, touching several other smaller ones, CCC, is struck by another sphere, B, in such a way as to make an impression upon each of its neighbors, it transfers its motion to them and remains at rest, as does also the sphere B. Now, without supposing that ether particles are spherical (for I do not see that this is necessary), we can nevertheless understand that this law of impulses plays a part in the propagation of the motion.

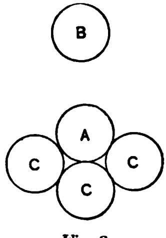

Fig. 3

Equality of size would appear to be a more necessary assumption, since otherwise we should expect the motion to be reflected on passing from a smaller to a larger particle, following the laws of percussion which I published some years ago. Yet, as will appear later, this equality is necessary not so much to make the propagation of light possible as to make it easy and intense. Nor does it appear improbable that the ether particles were made equal for a purpose so important as the transmission of light. This may be true, at least, in the vast region lying beyond our atmosphere and serving only to transmit the light of the sun and the stars.

I have now shown how we may consider light as propagated, in time, by spherical waves, and how it is possible that the speed of propagation should be as great as that demanded by experiment and by astronomical observation. It must, however, be added that although the ether particles are supposed to be in continual motion (and there is much evidence for this view), the gradual transmission of the waves is not thus interfered with. For it does not consist in a translation of these particles, but merely in a small vibration, which they are compelled to transmit to their neighbors in spite of their proper motion and their change of relative position.

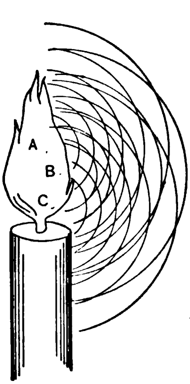

Fig. 4

But we must consider, in greater detail, the origin of these waves and the manner of their propagation from one point to another. And, first, it follows from what has already been said concerning the production of light that each point of a luminous body, such as the sun, a candle, or a piece of burning carbon, gives rise to its own waves, and is the centre of these waves. Thus if A, B, and C represent different points in a candle flame, concentric circles described about each of these points will represent the waves to which they give rise. And the same is true for all the points on the surface and within the flame. But since the disturbances at the centre of these waves do not follow each other in regular succession, we need not imagine the waves to follow one another at equal intervals; and if, in the figure, these waves are equally spaced, it is rather to indicate the progress which one and the same wave has made during equal intervals of time than to represent several waves having their origin at the same point.*

Nor does this enormous number of waves, crossing one another without confusion and without disturbing one another, appear unreasonable, for it is well known that one and the same particle of matter is able to transmit several waves coming from different, and even opposite, directions. And this is true not only in the case where the displacements follow one another in succession, but also where they are simultaneous. This is because the motion is propagated gradually. It is shown by the row of hard and equal spheres above mentioned. If we allow two equal spheres, A and D, to strike against the opposite sides of this row at the same instant, they will be observed to rebound each with the same speed that it had before collision, while all the other spheres remain at rest, although the motion has twice traversed the entire row. [*This evidently implies that the spheres A and D have equal speeds just before*

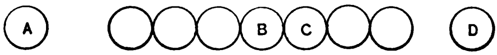

Fig. 5

collision.] If these two oppositely directed motions happen to meet at the middle sphere, B, or at any other sphere, say C, it will yield and spring back from both sides, thus transmitting both motions at the same instant.

* [*From this paragraph it would appear that Huygens had no conception of* trains *of light-waves. The experimental evidence for thinking that light-waves travel in trains seems first to have been furnished by Young. See pp.* 60, 61 *below. If, however, one prefers to interpret the colored rings of Newton in terms of the wave-theory, this experimental evidence may be ascribed to Newton.*]

But what is strangest and most astonishing of all is that waves produced by displacements and particles so minute should spread to distances so immense, as, for instance, from the sun or from the stars to the earth. For the intensity of these waves must diminish in proportion to their distance from the origin until finally each individual wave is of itself unable to produce the sensation of light. Our astonishment, however, diminishes when we consider that in the great distance which separates us from the luminous body there is an infinitude of waves which, although coming from different parts of the [*luminous*] body, are practically compounded into a single wave which thus acquires sufficient intensity to affect our senses. Thus the infinitely great number of waves which at any one instant leave a fixed star, as large possibly as our sun, unite to form what is equivalent to one single wave of intensity sufficient to affect the eye. Not only so, but each luminous point may send us thousands of waves in the shortest imaginable time, on account of the rapidity of the blows with which the particles of the luminous body strike the ether at these points. The effect of the waves would thus be rendered still more sensible.

In considering the propagation of waves, we must remember that each particle of the medium through which the wave spreads doe not communicate its motion only to that neighbor which lies in the straight line drawn from the luminous point, but shares it with all the particles which touch it and resist its motion. Each particle is thus to be considered as the centre of a wave. Thus if DCF is a wave whose centre and origin is the luminous point A, a particle at B, inside the sphere DCF, will give rise to its own individual [*secondary*] wave, KCL, which will touch the wave DCF in the point C, at the same instant in which the principal wave, originating at A, reaches the position DCF. And it is clear that there will be only one point of the wave KCL which will touch the wave DCF, *viz.*, the point which lies in the straight line from A drawn through B. In like manner, each of the other particles, *bbbb*, etc., lying within the sphere DCF, gives rise to its

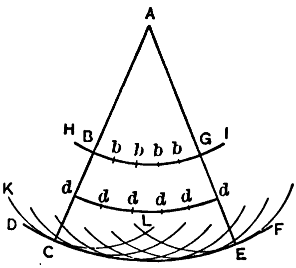

Fig. 6

own wave. The intensity of each of these waves may, however, be infinitesimal compared with that of DCF, which is the resultant of all those parts of the other waves which are at a maximum distance from the centre A.

We see, moreover, that the wave DCF is determined by the extreme limit to which the motion has travelled from the point A within a certain interval of time. For there is no motion beyond this wave, whatever may have been produced inside by those parts of the secondary waves which do not touch the sphere DCF. Let no one think this discussion mere hair-splitting. For, as the sequel will show, this principle, so far from being an ultra-refinement, is the chief element in the explanation of all the properties of light, including the phenomena of reflection and refraction. This is exactly the point which seems to have escaped the attention of those who first took up the study of light-waves, among whom are Mr. Hooke, in his *Micrographia,* and Father Pardies, who had undertaken to explain reflection and refraction on the wave-theory, as I know from his having shown me a part of a memoir which he was unable to finish before his death. But the most important fundamental idea, which consists in the principle I have just stated, is wanting in his demonstrations. On other points also his view is different from mine, as will some day appear in case his writings have been preserved.

Passing now to the properties of light, we observe first that each part of the wave is propagated in such a way that its extremities lie always between the same straight lines drawn from the luminous point.

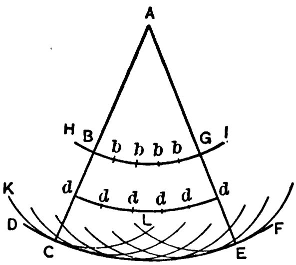

Fig. 6

For instance, that part of the wave BG, whose centre is the luminous point A, develops into the arc CE, limited by the straight lines, ABC and AGE. For although the secondary waves produced by the particles lying within the space CAE may spread to the region outside, nevertheless they do not combine at the same instant to produce one single wave limiting the motion and lying in the circumference CE which is their common tangent. This explains the fact that light, provided its rays are not reflected or refracted, always travels in straight lines, so that no body is illuminated by it unless the straight-line path from the source to this body is unobstructed.

Let us, for instance, consider the aperture BG as limited by the opaque bodies BH, GI; then, as we have just indicated, the light-waves will always be limited by the straight lines AC, AE. The secondary waves which spread into the region outside of ACE have not sufficient intensity to produce the sensation of light.

Now, however small we may make the opening BG, the circumstances which compel the light to travel in straight lines still remain the same; for this aperture is always sufficiently large to contain a great number of these exceedingly minute ether particles. It is thus evident that each particular part of any wave can advance only along the straight line drawn to it from the luminous point. And this justifies us in considering rays of light as straight lines.

From what has been said concerning the small intensity of the secondary waves, it would appear not to be necessary that all the ether particles be equal, although such an equality would favor the propagation of the motion. The effect of inequality would be to make a particle, in colliding with a larger one, use up a part of its momentum in an effort to recover. The secondary waves thus sent backward towards the luminous point would be unable to produce the sensation of light, and would not result in a primary wave similar to CE.

Another and more remarkable property of light is that when rays come from different, or even opposite, directions each produces its effect without disturbance from the other. Thus several observers are able, all at the same time, to look at different objects through one single opening; and two individuals can look into each other's eyes at the same instant.

If we now recall our explanation of the action of light and of waves crossing without destroying or interrupting each other, these effects which we have just described are readily understood, though they are not so easily explained from Descartes' point of view, *viz.*, that light consists in a continuous [*hydrostatic*] pressure which produces only a tendency to motion.

For such a pressure cannot, at the same instant, affect bodies from two opposite sides unless these bodies have some tendency to approach each other. It is, therefore, impossible to understand how two persons can look each other in the eye or how one torch can illuminate another.

CHAPTER II

ON REFLECTION

HAVING explained the effects produced by light-waves in a homogeneous medium, we shall next consider what happens when they impinge upon other bodies. First of all we shall see how reflection is explained by these same waves and how the equality of angles follows as a consequence. Let AB represent a plane polished surface of some metal, glass, or other substance, which, for the present, we shall consider as perfectly smooth (concerning irregularities which are unavoidable we shall have something to say at the close of this demonstration); and let the line AC, inclined to AB, represent a part of a light-wave whose centre is so far away that this part AC may be considered as a straight line. It may be mentioned here, once for all, that we shall limit our consideration to a single plane, *viz.*, the plane of the figure, which passes through the centre of the spherical wave and cuts the plane AB at right angles.

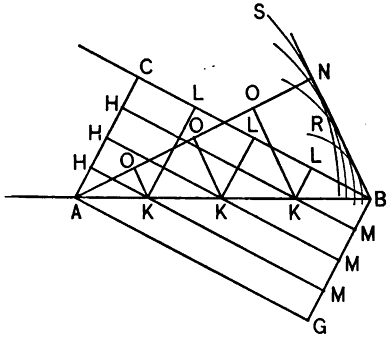

Fig. 7

The region immediately about C on the wave AC will, after a certain interval of time, reach the point B in the plane AB, travelling along the straight line CB, which we may think of as drawn from the source of light and hence drawn perpendicular to AC. Now in this same interval of time the region about A on the same wave is unable to transmit its entire motion beyond the plane AB; it must, therefore, continue its motion on this side of the plane to a distance equal to CB, sending out a secondary spherical wave in the manner described above. This secondary wave is here represented by the circle SNR, drawn with its centre at A and with its radius AN equal to CB.

So, also, if we consider in turn the remaining parts H of the wave AC, it will be seen that they not only reach the surface AB along the straight lines HK parallel to CB, but they will produce, at the centres K, their own spherical waves in the transparent medium. These secondary waves are here represented by circles whose radii are equal to KM—that is, equal to the prolongations of HK to the straight line BG which is drawn parallel to AC. But, as is easily seen, all these circles have a common tangent in the straight line BN, *viz.*, the same line which passes through B and is tangent to the first circle having A as centre and AN, equal to BC, as radius.

This line BN (lying between B and the point N, the foot of the perpendicular let fall from A) is the envelope of all these circles, and marks the limit of the motion produced by the reflection of the wave AC. It is here that the motion is more intense than at any other point, because, as has been explained, BN is the new position which the wave AC has assumed at the instant when the point C has reached B. For there is no other line which, like BN, is a common tangent to these circles, unless it be BG, on the other side of the plane AB. And BG will represent the transmitted wave only in case the motion occurs in a medium which is homogeneous with that above the plane.

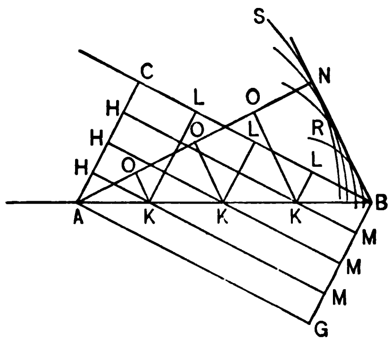

Fig. 7

If, however, one wishes to see just how the wave AC has gradually passed into the wave BN, he has only to use the same figure and draw the straight lines KO parallel to BN, and the straight lines KL parallel to AC. It is thus seen that the wave AC, from being a straight line, passes successively into all the broken lines OKL, and reassumes the form of a single straight line NB.

It is now evident that the angle of reflection is equal to the angle of incidence. For the right-angled triangles ABC and BNA have the side AB in common, and the side CB equal to the side NA, whence it follows that the angles opposite these sides are equal, and hence also the angles CBA and NAB. But CB, perpendicular to CA, is the direction of the incident ray, while AN, perpendicular to the wave BN, has the direction of the reflected ray. These rays are, therefore, equally inclined to the plane AB.

Against this demonstration it may be urged that while BN is the common tangent of the circular waves in the plane of this figure, the fact is that these waves are really spherical and have an infinitely great number of similar tangents, *viz.*, all straight lines drawn through the point B and lying in the surface of a cone generated by the revolution of a straight line BN about BA as axis. It remains to be shown, therefore, that this objection presents no difficulty; and, incidentally, we shall see that the incident and reflected rays each lie in one plane perpendicular to the reflecting plane.

I remark, then, that the wave AC, so long as it is considered merely a line, can produce no light. For a ray of light, however slender, must have a finite thickness in order to be visible. In order, therefore, to represent a wave whose path is along this ray, it is necessary to replace the straight line AC by a plane area, as, for instance, by the circle HC in the following figure, where the luminous point is supposed to be infinitely distant. From the preceding proof it is easily seen that each element of area on the wave HC, having reached the plane AB, will there give rise to its own secondary wave; and when C reaches the point B, these will all have a common tangent plane, *viz.*, the circle BN equal to CH. This circle will be cut through the centre and at right angles by the same plane which thus cuts the circle CH and the ellipse AB.

It is thus seen that the spherical secondary waves can have no common tangent plane other than BN. In this plane will be located more of the reflected motion than in any other, and it will therefore receive the light transmitted from the wave CH.

I have noted in the preceding explanation that the motion of the wave incident at A is not transmitted beyond the plane AB, at least not entirely. And here it is necessary to remark that, although the motion of the ether may be partly communicated to the reflecting body, this cannot in the slightest alter the speed of the propagation of the waves, which determines the angle of reflection. For, in any one medium, a slight disturbance produces waves which travel with the same speed as those

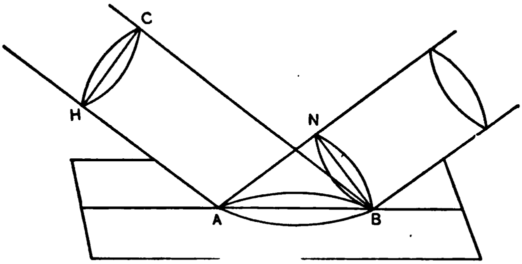

Fig. 8

due to a very great disturbance, a consequence of that property of elastic bodies concerning which we have spoken above, *viz.*, the time occupied in recovery is the same whether the compression be large or small. In every case of reflection of light from the surface of any substance whatever the angles of incidence and reflection are therefore equal, even though the body be of such a nature as to absorb a part of the motion delivered by the incident wave. And, indeed, experiment shows that among polished bodies there is no exception to this law of reflection.

We must emphasize the fact that in our demonstration there is no need that the reflecting surface be considered a perfectly smooth plane, as has been assumed by all those who have attempted to explain the phenomena of reflection. All that is called for is a degree of smoothness such as would be produced by the particles of the reflecting medium being placed one near another. These particles are much larger than those of the ether, as will be shown later when we come to treat of the transparency and opacity of bodies. Since, now, the surface consists thus of particles assembled together, the ether particles being above and smaller, it is evident that one cannot demonstrate the equality of the angles of incidence and reflection from the time-worn analogy with that which happens when a ball is thrown against a wall. By our method, on the other hand, the fact is explained without difficulty.

Take particles of mercury, for instance, for they are so small that we must think of the least visible portion of surface as containing millions, arranged like the grains in a heap of sand which one has smoothed out as much as possible; this surface for our purpose is equal to polished glass. And, though such a surface may be always rough compared with ether particles, it is evident that the centres of all the secondary waves of reflection which we have described above lie practically in one plane. Accordingly, a single tangent comes as near touching them all as is necessary for the production of light. And this is all that is required in our demonstration to explain the equality of angles without allowing the rest of the motion, reflected in various directions, to produce any disturbing effect.

CHAPTER III

ON REFRACTION

In the same manner that reflection has been explained by light-waves reflected at the surface of polished bodies, we propose now to explain transparency and the phenomena of refraction by means of waves propagated into and through transparent bodies, whether solids, such as glass, or liquids, such as water and oils. But, lest the passage of waves into these bodies appear an unwarranted assumption, I will first show that this is possible in more ways than one.

Let us imagine that the ether does penetrate any transparent body, its particles will still be able to transmit the motion of the waves just as do those of the ether, supposing them each to be elastic. And this we can easily believe to be the case with water and other transparent liquids, since they are composed of discrete particles. But it may appear more difficult in the case of glass and other bodies that are transparent and hard, because their solidity would hardly allow that they should take up any motion except that of their mass as a whole. This, however, is not necessary, since this solidity is not what it appears to us to be, for it is more probable that these bodies are composed of particles which are placed near one another and bound together by an external pressure due to some other kind of matter and by irregularity of their own configurations. For their looseness of structure is seen in the facility with which they are penetrated by the medium of magnetic vortices and those which cause gravitation. One cannot go further than to say that these bodies have a structure similar to that of a sponge, or of light bread, because heat will melt them and change the relative positions of their particles. We infer, then, as has been indicated above, that they are assemblages of particles touching one another but not forming a continuous solid. This being the case, the motion which these particles receive in the transmission of light is simply communicated from one to another, while the particles themselves remain tethered in their own positions and do not become disarranged among themselves. It is easily possible for this to occur without in any way affecting the solidity of the structure as seen by us.

By the external pressure of which I have spoken is not to be understood that of the air, which would be quite insufficient, but that of another and more subtle medium, whose pressure is exhibited by an experiment which I chanced upon a long while ago, namely, that water from which the air has been removed remains suspended in a glass tube open at the lower end, even though the air may have been exhausted from a vessel enclosing this tube.

We may thus explain transparency without assuming that bodies are penetrated by the luminiferous ether or that they contain pores through which the ether can pass. The fact, however, is not only that this medium penetrates ordinary bodies, but that it does so with the utmost ease, as indeed has already been shown by the experiment of Torricelli which we have cited above. When the mercury or the water leaves the upper part of the glass tube, the ether appears at once to take its place and transmit light. But following is still another argument for thinking that bodies, not only those which are transparent, but others also, are easily penetrable.

When light traverses a hollow glass sphere which is completely closed, it is evident that the sphere is filled with ether, just as is the space outside the sphere. And this ether, as we have shown above, consists of particles lying in close contact with each other. If, now, it were enclosed in the sphere in such a way that it could not escape through the pores of the glass, it would be compelled to partake of any motion which one might impress upon the sphere; consequently nearly the same force would be required to impress a given speed upon this sphere, lying upon a horizontal plane, as if it were filled with water, or possibly mercury. For the resistance which a body offers to any velocity one may wish to impart to it varies directly as the quantity of matter which the body contains and which is compelled to acquire velocity. But the fact is that the sphere resists the motion only in proportion to the amount of glass in it. Whence it follows that the ether within is not enclosed, but flows through the glass with perfect freedom.

Later we shall show, by this same process, that penetrability may be inferred for opaque bodies also.

A second and more probable explanation of transparency is to say that the light-waves continue on in the ether which always fills the interstices, or pores, of transparent bodies. For since it passes continuously and with ease, it follows that these pores are always full. Indeed, it may be shown that these interstices occupy more space than the particles which make up the body.

Now if it be true, as we have said, that the force required to impart a given horizontal velocity to a body is proportional to the mass of the body, and if this force be also proportional to the weight of the body, as we know by experiment that it is, then the mass of any body must be also proportional to its weight. Now we know that water weighs only $\frac{1}{14}$ part as much as an equal volume of mercury, therefore the substance of the water occupies only $\frac{1}{14}$ part of the space that encloses its mass. Indeed, it must occupy even a smaller fraction than this, because mercury is not so heavy as gold, and gold is a substance which is not very dense, since the medium of magnetic vortices and that which causes gravitation penetrate it with the utmost ease.

But it may be objected that if water be a substance of such small density, and if its particles occupy so small a portion of its apparent volume, it is very remarkable that it should offer such stubborn resistance to compression; for it has not been condensed by any force hitherto employed, and remains perfectly liquid while under pressure.

This is, indeed, no small difficulty. But it may nevertheless be explained by supposing that the very violent and rapid motion of the subtle medium which keeps water liquid also sets in motion the particles of which it is composed, and maintains this liquid state in spite of any pressure which has hitherto been applied.

If, now, the structure of transparent bodies be as loose as we have indicated, we may easily imagine waves penetrating the ether which fills the interstices between the particles. Not only so, but we can easily believe that the speed of these waves inside the body must be a little less on account of the small détours necessitated by these same particles. I propose to show that in this varying velocity of light lies the cause of refraction.

I will first indicate a third and last method by which we may explain transparency, namely, by supposing that the motion of the light-waves is transmitted equally well by the ether particles which fill the interstices of the body, and by the particles which compose the body, the motion being handed on from one to the other. A little later we shall see how beautifully this hypothesis explains the double refraction of certain transparent substances. Should one object that the particles of ether are much smaller than those of the transparent body, since the former pass through the intervals between the latter, and that consequently they would be able to communicate only a small portion of their momentum, we may reply that the particles of the body are composed of other still smaller particles, and that it is these secondary particles that take up the momentum from those of the ether.

Moreover, if the particles of the transparent body are slightly less elastic than are the ether particles, which we may reasonably suppose, it would still follow that the speed of the light waves is less inside the body than outside in the ether.

We have here, what appears to me, the manner in which light-waves are probably transmitted by transparent bodies. But there still remains the consideration of opaque bodies and the difference between these and transparent bodies, a question all the more interesting in view of the ease with which ether penetrates all bodies, a fact to which attention has already been directed, and which might lead one to think that all bodies should be transparent. For considering the hollow sphere, by which I have shown the open structure of glass and the ease with which ether passes through it, one would naturally infer the same penetrability as a property of metals and all other substances. Imagine the sphere to be of silver; it would then certainly contain luminiferous ether, because this substance, as well as air, would be present in it when the opening in the sphere was closed up. But when closed and placed upon a horizontal plane it would resist motion only in proportion to the amount of silver in it, showing as above that the enclosed ether does not acquire the motion of the sphere. Silver, therefore, like glass, is easily penetrated by ether. In between the particles of silver and of all other opaque bodies this substance is distributed continuously and abundantly; and, since it can transmit light, we are led to expect that these bodies should be as transparent as glass, which, however, is not the fact.

How, then, shall we explain their opacity? Are their constituent particles soft and built up of still smaller particles, and thus able to change shape when they are struck by ether particles? Do they thus damp out the motion and stop the propagation of the light-waves? This seems hardly possible; for if the particles of a metal were soft, how could polished silver and mercury reflect light so well? What seems to me more probable is that metallic bodies, which are almost the only ones that are really opaque, have interspersed among their hard particles some which are soft, the former producing reflection, the latter destroying transparency; while, on the other hand, transparent bodies are made up of only hard and elastic particles, which, together with the ether, propagate light-waves in the manner already indicated.

We pass now to the explanation of refraction, assuming, as above, that light-waves pass through transparent substances and in them undergo diminution of speed.

The fundamental phenomenon in refraction is the following, *viz.*, when any ray of light, AB, travelling in air, strikes obliquely upon the polished surface of a transparent body, FG, it undergoes a sudden change of direction at the point of incidence, B; and this change occurs in such a way that the angle CBE, which the ray makes with the normal to the surface, is less than the angle ABD, which the ray in air made with the same normal. To determine the numerical value of these angles, describe about the point B a circle cutting the rays AB, BC. Then the perpendiculars, AD, CE, let fall from these points of intersection upon the normal, DE, *viz.*, the sines of the angles ABD, CBE, bear to one another a certain ratio which, for any one transparent body, is constant for all directions of the incident ray. For glass this ratio is almost exactly $\frac{3}{2}$, while for water it is very nearly $\frac{4}{3}$, thus varying from one transparent body to another.

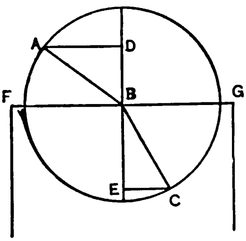

Fig. 9

Another property, not unlike the preceding, is that the refractions of rays entering and of rays emerging from a transparent body are reciprocal. That is to say, if an incident ray, AB, be refracted by a transparent body into the ray BC, so also will a ray, CB, in the interior of the body be refracted, on emergence, into the ray BA.

In order to explain these phenomena on our theory, let the straight line AB Fig. 10, represent the plane surface bounding a transparent body extending in a direction between C and N.

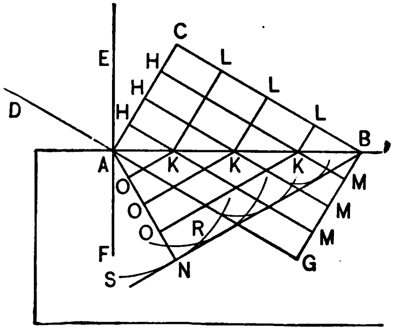

Fig. 10

By the use of the word plane we do not mean to imply a perfectly smooth surface, but merely such a one as was employed in treating of reflection, and for the same reason. Let the line AC represent a part of a light-wave whose source is so distant that this part may be treated as a straight line. The region C, on the wave AC, will, after a certain interval of time, arrive at the plane AB, along the straight line CB, which we must think of as drawn from the source of light, and which will, therefore, intersect AC at right angles. But during this same interval of time the region about A would have arrived at G, along the straight line AG, equal and parallel to CB, and, indeed, the whole of the wave AC would have reached the position GB, provided the transparent body were capable of transmitting waves as rapidly as the ether. But suppose that the rate of transmission is less rapid, say one-third less. Then the motion from the point A will extend into the transparent body to a distance which is only two-thirds of CB, while producing its secondary spherical wave as described above. This wave is represented by the circle SNR, whose centre is at A and whose radius is equal to $\frac{2}{3}$ CB. If we consider, in like manner, the other points H of the wave AC, it will be seen that, during the same time which C employs in going to B, these points will not only have reached the surface AB, along the straight lines HK, parallel to CB, but they will have started secondary waves into the transparent body from the points K as centres. These secondary waves are represented by circles whose radii are respectively equal to $\frac{2}{3}$ of the distances KM—that is, $\frac{2}{3}$ of the prolongations of HK to the straight line BG. If the two transparent media had the same ability to transmit light, these radii would equal the whole lengths of the various lines KM.

But all these circles have a common tangent in the line BN, *viz.*, the same line which we drew from the point B tangent to the circle SNR first considered. For it is easy to see that all the other circles from B up to the point of contact, N, touch, in the same manner, the line BN, where N is also the foot of the perpendicular let fall from A upon BN.

We may, therefore, say that BN is made up of small arcs of these circles, and that it marks the limits which the motion from the wave AC has reached in the transparent medium, and the region where this motion is much greater than anywhere else. And, furthermore, that this line, as already indicated, is the position assumed by the wave AC at the instant when the region C has reached the point B. For there is no other line below the plane AB, which, like BN, is a common tangent to all these secondary waves.

Accordingly, if one wishes to discover through what intermediate steps the wave AC reached the position BN, he has only to draw, in the same figure, the straight lines KO parallel to BN, and all the lines KL parallel to AC. He will thus see that the wave CA passes from a straight line into the successive broken lines LKO, reassuming the form of a straight line in the position BN. From what has preceded this will be so evident as to need no further explanation.

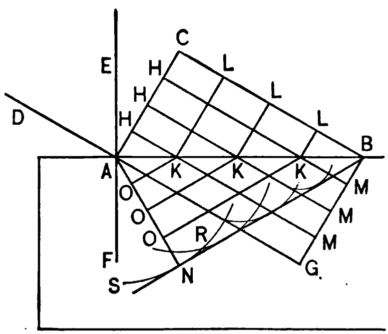

Fig. 10

If, now, using the same figure, we draw EAF normal to the plane AB at the point A, and draw DA at right angles to the wave AC, the incident ray of light will then be represented by DA; and AN, which is drawn perpendicular to BN, will be the refracted ray; for these rays are merely the straight lines along which the parts of the waves travel.

From the foregoing it is easy to deduce the principal law of refraction, *viz.*, that the sine of the angle DAE always bears a constant ratio of the sine of the angle NAF, whatever may be the direction of the incident ray, and that the ratio is the same as that which the speed of the waves in the medium on the side AE bears to their speed on the side AF.

For if we consider AB as the radius of a circle, the sine of the angle BAC is BC, and the sine of the angle ABN is AN. But the angles BAC and DAE are equal; for each is the complement of CAE. And the angle ABN is equal to NAF, since each is the complement of BAN. Hence the sine of the angle DAE is to the sine NAF as BC is to AN. But the ratio of BC to AN is the same as that of the speeds of light in the media on the side towards AE and the side towards AF, respectively; hence, also, the sine of the angle DAE bears to the sine of the angle NAF the same ratio as these two speeds of light.

In order to follow the refracted ray when the light-waves enter a body which transmits them more rapidly than the body from which they emerge (say in the ratio of 3 to 2), it is necessary only to repeat the same construction and the same demonstration which we have just been using, substituting, however, $\frac{3}{2}$ in place of $\frac{2}{3}$. And we find, by the same logical process, employing this other figure, that when the region C of the wave AC reaches the point B of the surface AB, the whole wave AC will have advanced to the position BN, such that the ratio of BC, perpendicular to AC, is to AN, perpendicular to BN, as 2 is to 3. The same ratio will also hold between the sine of the angle EAD and the sine of the angle FAN.

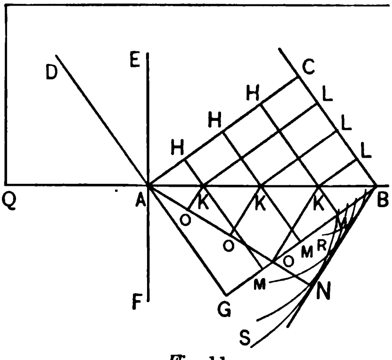

Fig. 11

The reciprocal relations between the refractions of a ray on entering and on emerging from one and the same medium is thus made evident. If the ray NA is incident upon the exterior surface AB, and is refracted into AD, then the ray DA on emerging from the medium will be refracted into AN.

We are now able to explain a remarkable phenomenon which occurs in this refraction. When the incident ray DA exceeds a certain inclination it loses its ability to pass into the other medium. Because if the angle DAQ or CBA is such that, in the triangle ACB, CB is equal to or greater than $\frac{2}{3}$ of AB, then AN, being equal to or greater than AB, can no longer form one side of the triangle ANB. Therefore the wave BN does not exist, and consequently there can be no line AN drawn at right angles to it. And thus the incident ray DA cannot penetrate the surface AB.

When the wave-speeds are in the ratio of 2 to 3, as in the case of glass and air, which we have considered, the angle DAQ must exceed 48° 11′ if the ray DA is to emerge. And when the ratio of speeds is that of 3 to 4, as is almost exactly the case in water and air, this angle DAQ must be greater than 41° 24′. And this agrees perfectly with experiment.

But one may here ask why no light penetrates the surface, since the encounter of the wave AC against the surface AB must produce some motion in the medium on the other side. The answer is simple, if we recall what has already been said. For although an infinite number of secondary waves may be started into the medium on the other side of AB, these waves at no time have a common tangent line, either straight or curved. There is thus no line which marks the limit to which the wave AC has been transmitted beyond the plane AB, nor is there any line in which the motion has been sufficiently concentrated to produce light.

In the following manner one may easily recognize the fact that, when CB is greater than $\frac{2}{3}$ AB, the waves beyond the plane AB have no common tangent. About the centres K describe circles having radii respectively equal to $\frac{3}{2}$ LB. These circles will enclose one another and will each pass beyond the point B.

It is to be noted that just as soon as the angle DAQ becomes too small to allow the refracted ray DA to pass into the other medium, the internal reflection which occurs at the surface AB increases rapidly in brilliancy, as may be easily shown by means of a triangular prism. In terms of our theory, we may thus explain this phenomenon: While the angle DAQ is still large enough for the ray DA to be transmitted, it is evident that the light from the wave-front AC will be concentrated into a much shorter line when it reaches the position BN. It will be seen also that the wave-front BN grows shorter in proportion as the angle CBA or DAQ becomes smaller, until finally, when the limit indicated above is reached, BN is reduced to a point. That is to say, when the region about C, on the wave AC, reaches B, the wave BN, which is the wave AC after transmission, is entirely compressed into this same point B ; and, in like manner, when the region about H has reached the point K the part AH is completely reduced to this same point K. It follows, therefore, that in proportion as the direction of propagation of the wave AC happens to coincide with the surface AB, so will be the quantity of motion along this surface.

Now this motion must necessarily spread into the transparent body and also greatly reinforce the secondary waves which produce internal reflection at the face AB, according to the laws of this reflection explained above.

And since a small diminution in the angle of incidence is sufficient to reduce the wave-front BN from a fairly large quantity to zero (for if this angle in the case of glass be 49° 11′, the angle BAN amounts to as much as 11° 21′; but if this same angle DAQ be diminished by one degree only, the angle BAN becomes zero and the wave-front BN is reduced to a point), it follows that the internal reflection occurs suddenly, passing from comparative darkness to brilliancy at the instant when the angle of incidence assumes a value which no longer permits refraction.

Now as to ordinary external reflection, *i. e.*, reflection which occurs when the angle of incidence DAQ is still large enough to allow the refracted ray to pass through the face AB, this reflection must be from the particles which bound the transparent body on the outside, apparently from particles of air and from others which are mixed with, but are larger than, the ether particles.

On the other hand, external reflection from bodies is produced by the particles which compose these bodies, and which are larger than those of the ether, since the ether flows through the interstices of the body.

It must be confessed that we here find difficulty in explaining the experimental fact that internal reflection occurs even where the particles of air can cut no figure, as, for instance, in vessels and tubes from which the air has been exhausted.

Experiment shows further that these two reflections are of almost equal intensity, and that in various transparent bodies this intensity increases directly as the refractive index. Thus we see that reflection from glass is stronger than that from water, while in turn that from diamond is stronger than that from glass.

I shall conclude this theory of refraction by demonstrating a remarkable proposition depending upon it, namely, that when a ray of light passes from one point to another, the two points lying in different media, refraction at the bounding surface occurs in such a way as to make the time required the least possible; and exactly the same thing occurs in reflection at a plane surface. M. Fermat discovered this property of refraction, believing with us and in opposition to M. Descartes that light travels more slowly through glass than through air. But, besides this, he assumed what we have just proved from the fact that the velocities in the two media are different, *viz.*, that the ratio of the sines is a constant; or, what amounts to the same thing, he assumed, besides the different velocities, that the time employed was a minimum; and from this he derived the constancy of the sine ratio.

His demonstration, which may be found in his works and in the correspondence of M. Descartes, is very long. It is for this reason that I here offer a simpler and easier one.

Let KF represent a plane surface; imagine the point A in the medium through which the light travels more rapidly, say air; the point C lies in another, say water, in which the speed of light is less. Let us suppose that a ray passes from A, through B, to C, suffering refraction at B, according to the law above demonstrated; or, what is the same thing, having drawn PBQ perpendicular to the surface, the sine of the angle ABP is to the sine of the angle CBQ in the same ratio as the speed of light in the medium

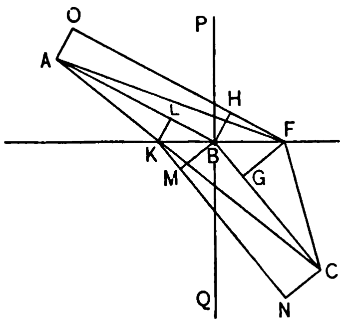

Fig. 12

containing A is to the speed in the medium containing C. It remains to show that the time required for the light to traverse AB and BC taken together is the least possible. Let us assume that the light takes some other path, say AF, FC, where F, the point at which refraction occurs, is more distant than B from A. Draw AO perpendicular to AB, and FO parallel to BA; BH perpendicular to FO, and FG perpendicular to BC. Since, now, the angle HBF is equal to PBA, and the angle BFG is equal to QBC, it follows that the sine of the angle HBF will bear to the sine of BFG the same ratio as the speed of light in the medium A bears to the speed in the medium C. But if we consider BF the radius of a circle, then sines are represented by the lines HF, BG. Accordingly, the lines HF, BG are in the ratio of these speeds. If, therefore, we imagine OF to be the incident ray, the time of passage from H to F will be the same as the time of passage from B to G in the medium C.

But the time from A to B is equal to the time from O to H. Hence the time from O to F is the same as the time from A to G, *via* B. Again, the time along FC is greater than the time along GC; and hence the time along the route OFC is greater than that along the path ABC. But AF is greater than OF; hence, *a fortiori*, the time along AFC is greater than that along ABC.

Let us now assume that the ray passes from A to C by the route AK, KC, the point of refraction, K, being nearer to A than is B. Draw CN perpendicular to BC; KN parallel to BC; BM perpendicular to KN; and KL perpendicular to BA.

Here BL and KM represent the sines of the angles BKL and KBM—that is, the angles PBA and QBC; and hence they are in the same ratio as the speeds of light in the media A and C respectively. The time, therefore, from L to B is equal to the time from K to M; and, since the time from B to C is equal to the time from M to N, the time by the path LBC is the same as the time *via* KMN. But the time from A to K is greater than the time from A to L, and, therefore, the time by the route AKN is greater than the route ABC.

Not only so, but since KC is greater than KN, the time by the path AKC will be so much the greater than by the path ABC. Hence follows that which was to be proved, namely, that the time along the path ABC is the least possible.